Quantifying Reduced-Form Evidence on Collateral Constraints

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Abstract

This paper quantifies the aggregate effects of financing constraints. We start from a standard dynamic model of investment with collateral constraints. In contrast to the existing quantitative literature, our estimation does not target the mean leverage ratio to identify the scope of financing frictions. Instead, we use a reduced-form coefficient from the recent corporate finance literature that connects exogenous shocks to debt capacity to corporate investment. We embed the estimated model in a simple general equilibrium framework and find that, relative to first-best, collateral constraints induce output losses of 7.1%, and TFP (misallocation) losses of 1.4%. We show that these estimated economic losses tend to be more robust to misspecification bias than estimates obtained by targeting leverage.

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There is an accumulating body of evidence showing the causal effect of financing frictions on *firm-level* outcomes. For instance, Lamont (1997) shows that reduction in oil prices lead non-oil subsidiaries of oil companies to reduce capital expenditures; Rauh (2006) exploits nonlinear funding rules for defined benefit pension plans to identify the role of internal resources on corporate investment; Chaney et al. (2012) and Gan (2007) use variations in local house prices as shocks to firms collateral value and show that collateral values affect investment; Chodorow-Reich (2013) combines the default of Lehman Brothers with the stickiness in banking relationships to show how bank lending frictions distort labor demand; Lian and Ma (2019) and Greenwald (2019) show that exogenously binding covenants distort investment.[1] While this literature safely rejects the null hypothesis that firms are not financially constrained, it provides little guidance to quantify the economic importance of financial constraints that derive from their estimates. The objective of this paper is to fill this gap in the literature.

We focus on a pervasive source of financing friction – collateral constraints – and build our quantitative analysis around a reduced-form estimate showing the significant effect of collateral values on firm-level investment, which we denote β throughout the paper. Gan (2007) and Chaney et al. (2012) show that corporate investment of firms owning real estate assets responds to fluctuations in local real estate prices relative to firms renting properties. These estimates reject the null hypothesis of no financing friction. However, *per se*, these estimates do not tell us whether these constraints matter *quantitatively*, both in partial and in general equilibrium. To answer this question, we need to evaluate how corporate behavior and aggregate efficiency would change *in the absence of financing constraint*, a counterfactual that is hard to measure in the data.

Our approach consists of building a structural model of financially constrained firms and requiring it to match β, the reduced-form impact of real estate collateral on investment. The model builds on the standard neoclassical model of investment with adjustment costs (Jorgenson, 1963; Lucas, 1967; Hayashi, 1982), to which we add a collateral constraint and restrictions on equity issuances. Collateral is made of capital and real estate, whose price fluctuates randomly.[2] Higher real estate prices increase debt capacity and investment, thus mimicking the behavior observed in the data.

[1]Other contributions include, but are not limited to, Banerjee and Duflo (2014), Lemmon and Roberts (2010), Faulkender and Petersen (2012), Zia (2008), Zwick and Mahon (2017), Benmelech et al. (2011), Benmelech et al. (2017), ...

[2]While we do not explicitly provide a micro-foundation for the collateral constraint, it emanates naturally from limited enforcement models as in Hart and Moore (1994).

Our estimation procedure explicitly targets the reduced-form impact of variations in local real estate prices on investment as a key moment in the estimation. In the estimated model, firms behave as if they could only pledge about 25% of their capital and real estate collateral. In the cross-section of firms, we observe that constrained firms are more likely to be growth firms and low productivity firms. Consistent with the literature on misallocation (Hsieh and Klenow, 2009), constrained firms have a high marginal revenue product of capital (MRPK) or a high Tobin's Q.

In a second step, the estimated model is nested in a simple general equilibrium framework where firms compete for customers, workers, and capital goods. To assess the economic magnitudes implied by the reduced-form estimates, we simulate two economies: one in which firms face the estimated collateral constraint, and a counterfactual economy where firms are unconstrained financially. We then compare these two economies.[3] We find an aggregate output loss of 7%. Out of these 7%, 1.4% come from lower TFP, i.e. *misallocation* of labor and capital across firms (Hsieh and Klenow, 2009; Moll, 2014; Midrigan and Xu, 2014), but most of the output loss comes from lower factor use: In the constrained economy, the aggregate capital stock is lower by 13.7% and employment is lower by 2.4%.[4]

While we are definitely not the first to measure the costs of financial constraints, our paper differs from earlier studies in an important way: To identify financing constraints, we target a moment that the empirical corporate finance literature directly ties to financing constraints. Papers in the existing quantitative literature (e.g. Buera et al. (2011), Midrigan and Xu (2014)) typically estimate financing frictions using the average debt to capital ratio (henceforth "leverage"). The intuition is simple: High leverage means strong debt capacity and therefore loose financing constraints. The empirical corporate finance literature, however, does not use leverage as a measure of financing constraints. It has developed better-identified measures but does not make any quantitative inference on the economic implications of these measures. Our paper fills this gap between the quantitative and the empirical literature on financing constraints.

Does it make a difference? In Section 5 we show that estimates of losses from financing constraints obtained by targeting leverage tend to be more vulnerable to misspecification bias.

[3]Of course, a counterfactual in which there are no financing constraints at all is certainly not policy-relevant, but it serves as a useful benchmark to measure of how binding financing constraints are.

[4]In line with the macroeconomic literature, we formally quantify the *cost* of financing frictions, but not their potential *benefit*. We model collateral constraints in a reduced-form way and do not take a stance on whether the rationale behind these collateral constraints is efficient in a second-best sense.

We first follow Andrews et al. (2017) and compute the local *sensitivity* of output and TFP losses to both average leverage and the reduced-form coefficient β. This sensitivity is twice larger for leverage than for β. This diagnostic suggests that estimates targeting leverage are potentially more exposed to misspecification bias (Andrews et al. (2017)). However, this evidence is only suggestive: Actual misspecification bias also depends on how much leverage and β are themselves affected by misspecification. We investigate the effect of misspecification on these moments in two different ways.

First, we consider sources of misspecification that arise purely from *measurement issues*. We assume our model is correctly specified but the moments we use for estimation are mismeasured. We consider several sources of mismeasurement highlighted in the corporate finance literature. For instance, the capital stock may be mismeasured because of operating leases (Rampini and Eisfeldt, 2009), intangibles (Peters and Taylor, 2017), or because economic depreciation is smaller than accounting depreciation. Similarly, secured debt may be mismeasured in the presence of operating leases (Rampini and Eisfeldt, 2009), account payables (Barrot, 2016), or unsecured debt (Benmelech et al., 2019). We estimate average leverage and β in our sample using six alternative measures of debt and capital that account for such measurement issues. We find that average leverage is typically more sensitive to these alternative measures than the reduced-form moment β. Combined with the lower sensitivity of TFP and output losses to β, this finding suggests that estimations targeting leverage may result in higher misspecification bias than estimations targeting β.

Second, we consider the opposite case: the moments are correctly measured, but the model is misspecified. We simulate a large number of datasets generated by alternative models that deviate from our baseline model in several dimensions, including the presence of intangible capital, unobserved debt capacity, or unsecured debt capacity. Because we consider potentially large deviations, we depart from Andrews et al. (2017) and adopt a global approach to measuring misspecification bias. Precisely, we estimate our baseline (hence misspecified) model using these simulated datasets in two ways: a first estimation targets leverage; a second estimation targets the reduced-form coefficient β. Since we know the data-generating process, we can directly measure misspecification bias for each of these estimations by comparing the estimated TFP or output loss to their true value. Our analysis of 4,000 alternative models reveals that estimates obtained by targeting the reduced-form coefficient β exhibit, on average, a lower misspecification bias than those obtained by targeting leverage.

Related Literature. Our paper contributes to the macroeconomic literature on the aggregate effects of financial frictions (for example Buera et al. (2011), Jermann and Quadrini (2012), Kahn and Thomas (2013)). Our model is closest to Midrigan and Xu (2014), with the notable difference that firms can use real estate as collateral in addition to physical capital. In these papers, the real effects of financing constraints are typically estimated by targeting a moment related to corporate leverage in the economy. The intuition behind that strategy is that a high corporate leverage implies that firms can easily pledge capital to lenders so that financial constraints are loose. Our contribution to this literature is to use a moment coming from the reduced-form literature, which directly connects exogenous shocks to debt capacity to real investment decisions. We argue that, as a targeted moment, the leverage ratio is more sensitive to measurement issues, which may arise, for instance, in the presence of off-balance sheet debt and capital. In turn, we show that such measurement error may materially affect estimates of the costs of financial constraints. In Monte-Carlo simulations, we show that inference based on our preferred reduced-formm moment is more robust to this type of misspecifications.

Our paper also contributes to the structural corporate finance literature (see Strebulaev and Whited (2012) for a review). Our model is closest to Hennessy and Whited (2007), which we adapt by adding real estate collateral. We contribute to this literature in several ways. First, instead of mean leverage, we target the reduced-form effect of shocks to collateral value on investment. Second, we nest our investment model into a general equilibrium model, which allows us to account for general equilibrium effects in our counterfactuals. In contrast, this literature typically only considers partial equilibrium counterfactuals. This is a crucial modification since, at equilibrium, constrained firms compete for resources and consumers with unconstrained firms. Third, our focus on a policy function (how investment reacts to debt capacity) as a targeted moment brings us close to a tradition in the structural literature (see Bajari et al. (2007), Bazdresch et al. (2018)).

Section 1 calculates the key moments used in our inference on financial constraints. Section 2 presents our formal model of firm dynamics with collateral constraints. Section 3 structurally estimates the model using US firm-level data. Section 4 describes and implements the general equilibrium analysis and our counterfactual measure of the aggregate effects of collateral constraints. Section 5 compares misspecification bias that arises in estimations that target leverage vs. the reduced-form coefficient β.

1 Real Estate Collateral and Investment

In this paper, we base our estimation on a measure of financing constraints coming from the reduced-form literature. We estimate how real estate collateral affects investment as in Chaney et al. (2012). The construction of the data is detailed in that paper. The dataset is a panel of publicly listed firms from 1993 to 2006 extracted from COMPUSTAT. We require that these firms supply information about the accounting value and cumulative depreciation of land and buildings (items ppenb, ppenli, dpacb, dpacli) in 1993. We combine this information with office prices in the MSA where headquarters are located to obtain a measure of the market value of firms' real estate holdings normalized by the previous year property, plant, and equipment. We call this measure REValue_{it} for firm i at date t.

We then follow the preferred specification of Chaney et al. (2012) and run the following regression:

$$\frac{i_{it}}{k_{it-1}} = a + \beta . \frac{\text{REValue}_{it}}{k_{it-1}} + \text{Offprice}_{it} + controls_{it} + \nu_{it}, \tag{1}$$

where i_{it} is investment (item capx), k_{it-1} is the lagged stock of productive capital (item ppent). Offprice_{it} is an index for office prices in the MSA where firm i's headquarters are located. This index is available from Global Real Analytics for 64 MSAs. We include the same controls as in their Table 5, column 5, i.e. firm- and year-specific fixed effects, as well as firm-level controls interacted with real estate prices. We cluster error terms ν_{it} at the firm level. We are interested in the reduced-form moment β, the estimated impact of real estate value on investment.

The only difference with Chaney et al. (2012) is that we add about 900 MSA $\times$ year fixed effects, which forces the identification on the comparison between owners and renters within MSA-years. This leaves the estimate of their Table 5, column 5, unchanged at 0.06. The t-stat weakens somewhat but remains high at 6.1 in this highly saturated specification. This moment suggests that every \$1 of real estate appreciation translates into \$0.06 of additional investment. The rest of the paper quantifies this reduced-form estimate in terms of firm-level financial friction and aggregate efficiency and output losses.

2 The model

In this section, we lay out our model of investment dynamics under collateral constraints. The economy is populated by heterogeneous, financially constrained firms, which combine capital and labor to produce differentiated goods. Those differentiated goods are then combined into a final good, consumed by a representative consumer, and used as a capital good.

2.1 Production technology and demand

The firm-level model is close to Hennessy and Whited (2007): it includes a tax shield for debt and a cost of equity issuance. It is also close to Liu et al. (2013): firms face a collateral constraint. The firm's shareholder is risk-neutral and her time discount rate is r. Firm i produces output q_{it} combining capital k_{it} and efficiency units of labor l_{it} into a Cobb-Douglas production function with capital share α

$$q_{it} = F\left(e^{z_{it}}, k_{it}, l_{it}\right) = e^{z_{it}}\left(k_{it}^{\alpha} l_{it}^{1-\alpha}\right), \tag{2}$$

with z_{it} the firm's log total factor productivity following an AR(1) process:

$$z_{it} = \rho z_{it-1} + \eta_{it},$$

and σ^2 the variance of the innovation η_{it}. The firm faces a downward sloping demand curve with constant elasticity $\phi > 1$,

$$q_{it} = Q p_{it}^{-\phi}. \tag{3}$$

Q is aggregate spending and will be determined in equilibrium (see Section 4).

Labor is fully flexible. w is the wage, also determined in equilibrium. As labor is a static input, the total profits of the firm, net of labor input, and before taxes, is

$$\pi\left(z_{it}; k_{it}\right) = \max_{l_{it}}\{p_{it}q_{it} - w l_{it}\} = bQ^{1-\theta} w^{-\frac{(1-\alpha)}{\alpha}\theta} e^{\frac{\theta}{\alpha} z_{it}} k_{it}^{\theta}, \tag{4}$$

with b a scaling constant and $\theta \equiv \frac{\alpha(\phi-1)}{1+\alpha(\phi-1)} < 1$.

2.2 Input dynamics

While labor is a static input, capital is not. Capital accumulation is subject to depreciation, time to build, and adjustment costs. Gross investment i_{it} is given by

$$k_{it+1} = k_{it} + i_{it} - \delta k_t, \tag{5}$$

where δ is the depreciation rate. In period t, investing i_{it} entails a convex cost of $\frac{c}{2}\frac{i_{it}^2}{k_{it}}$. Additionally, the firm pays in period t for capital that will only be used in production in period $t+1$: This one period time to build for capital is conventional in the macro literature (Hall, 2004; Bloom, 2009) and acts as an additional adjustment cost. Introducing adjustment costs to capital is important in our estimation exercise since they generate patterns similar to financing constraints and could thus be a natural confounding factor in our estimation procedure. For instance, adjustment costs make capital vary less than firm output, which generates a natural dispersion in capital productivities, mimicking financing constraints (Asker et al., 2014). As we will show below, using the reduced-form moments presented in Section 1 allows us to identify both frictions separately.

We do not, however, include fixed adjustment costs to our model, a choice also made by Gourio and Kashyap (2007): Our estimation targets firm-level data at an annual frequency, for which investment is not very lumpy. In our sample, only 4% of the observations have an investment rate smaller than 2% of capital.[5]

2.3 Financing frictions and capital structure

The firm finances investment out of retained earnings, debt, and equity issuance to outside investors. d_{it} is net debt, so that $d_{it} < 0$ means that the firm holds cash. As is standard in the structural corporate finance literature (Hennessy and Whited, 2005), we only consider short-term debt contracts with a one period maturity. We set up the model so that debt is risk-free and pays an interest rate r[6] – determined in equilibrium in Section 4. For an amount d_{it} of debt issued at date t, the firm commits to repay $(1+r)d_{it+1}$ at date $t+1$. Finally, the interest rate the firm receives on cash is lower than the interest rate it has to pay on its debt: If the firm has negative net debt, it receives a positive cash inflow of $-(1+(1-m)r)d_{it+1}$ with $0 < m < 1$.

Consistently with the corporate finance literature, we also assume firms' profits net of interest payments and capital depreciation, δk_{it}, are taxed at rate τ. This tax rate applies both to negative and positive income so that firms receive a tax credit

[5]To compute the investment rate, we divide item capx by lagged item ppent

[6]While this risk-free interest rate could in principle be time-varying, i.e. r_t, it will always be constant in our model, pinned down by the consumer's Euler equation with no aggregate risk, and we thus omit the t subscript for simplicity.

when their accounting profits are negative.[7] Other papers make alternative assumptions to make debt attractive to firms, either by assuming that debt holders are intrinsically more patient than shareholders, or that shareholders with log utility seek to smooth consumption as in Midrigan and Xu (2014). Finally, note that all tax proceeds are rebated to the representative consumer (see Section 4).

The financing frictions come from the combination of two constraints. First, equity issuance is costly: If pre-issuance cash-flows are x, cash-flows net of issuance costs are given by:

$$G(x) = x\left(1 + e1_{x<0}\right)$$

where $e > 0$ parameterizes the cost of equity issuance. Second, firms face a collateral constraint, which emanates from limited enforcement (Hart and Moore, 1994). We follow Liu et al. (2013) and adopt the following specification for the collateral constraint:

$$(1 + r)d_{it+1} \leq s\left((1 - \delta)k_{it+1} + \mathbb{E}[p_{t+1}|p_t] \times h\right). \tag{6}$$

The total collateral available to the creditor at the end of period $t + 1$ consists of depreciated productive capital $(1 - \delta)k_{it+1}$ and real estate assets with value $p_{t+1}h$. We assume log p_t to be a discretized AR(1) process. s, the share of the collateral value realized by creditors, captures the quality of debt enforcement, but also the extent to which collateral can be redeployed and sold.[8]

In assuming that the quantity of real estate h is the same across firms and time, we abstract from issues related to real estate ownership heterogeneity, which is an important limitation of this paper. In reality, we recognize that firms' decision to buy or lease real estate assets can potentially depend on expected productivity, investment opportunities, local factor prices, and financing constraints. We leave the analysis of how the endogeneity of real estate ownership affects current investment decisions for future research and focus this paper on measuring and aggregating financial frictions given the observed levels of real estate ownership in the data.

[7] As a result, debt is tax-free, which creates an incentive for firms to increase their leverage. This assumption marginally simplifies exposition and is consistent with several features of the tax code such as the presence of tax loss carry-forwards, but is not crucial for our results.

[8] The formulation of the collateral using the expected future value of collateral is standard in macroeconomics. It can be justified as an optimal contract in a set-up where (1) the firm has the entire bargaining power in its relationship with creditors (2) it cannot commit not to renegotiate the debt contract at the end of period t and (3) collateral can only be seized at the end of period $t + 1$.

2.4 The optimization problem

The firm is subject to a death shock with probability D, but infinitely lived otherwise. Every period, physical capital and debt are chosen optimally to maximize a discounted sum of per period cash flows, subject to the financing constraint. The firm takes as given its productivity, local real estate prices, and forms rational expectations for future productivities and real estate prices.

Define as $V(S_{it}; X_{it})$ the value of the discounted sum of cash flows given the exogenous state variables $X_{it} = \{z_{it}, p_t\}$ and the past endogenous state variables $S_{it} = \{k_{it}, d_{it}\}$. Shareholders are assumed to be perfectly diversified so their discount rate is the same as risk-free debt r.

This value function V is the solution to the following Bellman equation,

$$\begin{cases} V(S_{it}; X_{it}) & = \max_{S_{it+1}} \left\{ CF + \frac{1-D}{1+r} \mathbb{E}\left[V(S_{it+1}; X_{it+1}) \,|X_{it}\right] + \frac{D}{1+r}\left(k_{it+1} - (1+\tilde{r}_{it})d_{it+1}\right) \right\} \\ \text{s.t.} & (1+r)d_{it+1} \le s\left((1-\delta)k_{it+1} + \mathbb{E}[p_{t+1}|p_t] \times h\right) \\ \text{with:} & CF = G\left(\pi(z_{it}; k_{it}) - i_{it} - \frac{c}{2}\frac{i_{it}^2}{k_{it}} + d_{it+1} - (1+\tilde{r}_{it})d_{it} - \tau\left(\pi(z_{it}; k_{it}) - \tilde{r}d_{it} - \delta k_{it}\right)\right) \\ & i_{it} = k_{it+1} - (1-\delta)k_{it} \\ & \tilde{r}_{it} = r \text{ if } d_{it} > 0 \text{ and } (1-m)r \text{ if } d_{it} \le 0 \end{cases} \tag{7}$$

where the second term in the maximand ($\frac{D}{1+r}\left(k_{it+1} - (1+\tilde{r}_{it})d_{it+1}\right)$) corresponds to the shareholder's payoff in case of firm death. This term avoids a bias towards borrowing. If bankers could recover capital when a firm exits, shareholders would have an incentive to borrow more to transfer value from states of nature where they cannot consume to states where the firm survives. By assuming that shareholders receive the remaining capital when the firm exits, we ensure that this risk-shifting behavior does not drive the capital structure decisions of firms in our model.

Aggregate demand Q and the real wage w are equilibrium variables that the firms take as given when optimizing inputs. Given the absence of aggregate uncertainty and the steady-state assumption, they are fixed over time. Due to downward-sloping demand, firms have an optimal scale of production. A firm initially below this level accumulates capital, but only gradually because of convex adjustment costs and time to build. Finally, spending on adjusting capital is bounded by the collateral constraint. When the value of a firm's real estate assets increases, the collateral constraint is relaxed, and the firm finances more of the cost of adjusting towards its desired scale. This generates the response of investment to shocks to collateral value documented in Section 1.

3 Structural Estimation

3.1 Estimation procedure

We estimate the key parameters of the model via a Simulated Method of Moments. The entire procedure is described in detail in Appendix A. We look for the set of parameters $\hat{\mathbf{\Omega}}$ such that model-generated moments $\mathbf{m}(\hat{\mathbf{\Omega}})$ on simulated data fit a predetermined set of data moments $\mathbf{m}$. If we could solve the model analytically, we could just invert the system of equations given by model-based moments. Because our model does not have an analytic solution, we need to use indirect inference to perform the estimation. Such inference is done in two steps:

1. For a given set of parameters, we solve the Bellman problem (7) numerically and obtain the policy function $S_{it+1} = (d_{it+1}, k_{it+1})$ as a function of $S_{it} = (d_{it}, k_{it})$ and exogenous variables $X_{it} = (z_{it}, p_t)$. We discretize the state space (S, X) into a grid that is as fine as possible to minimize numerical errors in the presence of hard financing constraints. This is critical: A 1-2% numerically generated error would be too large to quantify aggregate effects of this order of magnitude. Solving the model repeatedly to estimate our structural parameters would not be feasible on a conventional CPU (several hours per iteration), so we use a GPU instead (a few minutes per iteration), as described in Appendix A.1.

2. Our parameter estimates minimize the distance from simulated to data moments,

$$\widehat{\mathbf{\Omega}} = \arg\min_{\mathbf{\Omega}} \left(\mathbf{m} - \widehat{\mathbf{m}}\left(\mathbf{\Omega}\right)\right)' \mathbf{W} \left(\mathbf{m} - \widehat{\mathbf{m}}\left(\mathbf{\Omega}\right)\right),$$

where the weighting matrix $\mathbf{W}$ is the inverse of the variance-covariance matrix of data moments. Standard errors are calculated by bootstrapping. Appendix A.2 describes how we escape the many local minima of our objective function, and how we correct for both estimation and simulation errors.

3.2 Predefined and Estimated Parameters

The model has 15 parameters. We calibrate 10 of them using estimates from the literature or the data and estimate the five remaining ones.

Predefined parameters. Our 10 calibrated parameters are as follows. We set the capital share $\alpha = 1/3$ from Bartelsman et al. (2013) and the demand elasticity $\phi = 6.7$, within the range of Broda and Weinstein (2006) (18% mark-ups in the absence of adjustment costs). Log real estate prices $\log p_t$ follow a discretized AR(1) process.

We estimate this AR(1) process on de-trended logged real estate prices and find a persistence $.62$ and innovation volatility $.06$. Both AR(1) processes for $\log z_t$ and $\log p_t$ are discretized using Tauchen's method. The rate of obsolescence of capital is set at $\delta = 6\%$ as in Midrigan and Xu (2014). The risk-free borrowing rate r is fixed at 3%, while the lending rate is set to $(1 - m)r = 2\%$. We fix the death rate D to 8% which corresponds to the turnover rate of firms in our data. We set the corporate tax rate τ at 33%. Since one may argue that effective tax rates may differ from statutory ones, we explore later the robustness of our inference with respect to the tax rate. The amount of real estate collateral h is set to match the average ratio of real estate to capital h/k_t exactly (0.14 for the average ratio of real estate, COMPUSTAT item land + building in 1993, to total assets, COMPUSTAT item at). Finally, we normalize $w = 0.03$ and $Q = 1$ for the estimation. This normalization is done without loss of generality because, in this model, the couple (Q, w) used for estimation has no effect on estimated structural parameters and aggregate outcomes (Sraer and Thesmar, 2018).[9] Q and w will be endogenously determined in general equilibrium in our counterfactual analyses (see Section 4).

Estimated parameters. We estimate 5 parameters: The persistence ρ and innovation volatility σ of log productivity, the collateral parameter s, the adjustment cost c and the cost of equity issuance e.

3.3 Data Moments

We compute the moments on the COMPUSTAT sample described in Section 1. We describe them here with a short heuristic discussion on identification. In the next section, we discuss identification more systematically and show how simulated moments vary with parameters.

First, in the spirit of Midrigan and Xu (2014), we use the short- and long-term volatility of output to estimate the persistence and volatility of log sales. In our sample, the volatility of change in log sale ($\log \text{sales}_{it} - \log \text{sales}_{it-1}$, COMPUSTAT item:

[9] The intuition is the following (Sraer and Thesmar, 2018): In partial equilibrium (i.e. for (Q, w) fixed), the level of (Q, w) scales up all firm outcomes by the same constant factor (let's call it φ). As a result, the simulated moments do *not* depend on the level of (Q, w) chosen. Ratios like leverage, investment rate, real estate value divided by capital, are unaffected when scaling by φ. The variance of log sales isn't either. This ensures that the set of structural parameters we estimated via SMM does not depend on the level of (Q, w) chosen. Besides, aggregate outcomes are also insensitive to the level of (Q, w) chosen in estimation. In our macro framework, the CES aggregator that we (like many others in the macro-finance literature) use ensures that aggregate output and TFP only depends on the distribution of MRPKs $\log \frac{py}{k}$. MRPKs are ratios and thus insensitive to the level of (Q, w) used to simulate the economy.

sale) equals 0.327. The volatility of 5-year change in log sales ($\log \text{sales}_{it} - \log \text{sales}_{it-5}$) equals 0.912. The fact that 5-year growth is less than 5 times more volatile than 1-year growth contributes to the identification of the persistence parameter. Targeting these two moments instead of directly matching the persistence coefficient of log sales makes our estimation less sensitive to short panel bias. Indeed, with firm fixed effects, and even though in our panel firms are present about 9 years on average, a fixed-effect estimator of persistence is strongly downward biased (Nickell, 1981). Targeting variances of log changes at various horizons allows us to bypass this problem.

Second, we use the autocorrelation of investment to identify quadratic adjustment costs (Bloom, 2009). For each firm in our panel we compute the ratio $\frac{i_{it}}{k_{it-1}}$ of capital expenditures (COMPUSTAT item: capx) to *lagged* capital stock (COMPUSTAT item: ppent). The correlation between $\frac{i_{it}}{k_{it-1}}$ and $\frac{i_{it-1}}{k_{it-2}}$ in our data is 0.12. Adjustment costs compel the firm to smooth its investment policy in response to a productivity shock (Asker et al., 2014). Financing frictions add to this smoothing motive.

Third, we use a direct measure of financing constraints to identify the collateral constraint parameter s: the sensitivity of investment to real estate value, which corresponds to the reduced-form moment β estimated from equation (1) in Section 1. This regression coefficient is directly related to financing frictions: Under our identifying assumption, this coefficient would be statistically insignificant absent financing frictions. While one can reject the *absence* of financing frictions if this coefficient is positive, its precise level does not map one for one into any structural parameter of our model. It does however allow us to identify the level of financing frictions through indirect inference. This is one of the main contributions of our paper: We show how to bridge the gap between the reduced-form corporate finance literature and the structural finance and macroeconomics literature.

In lieu of this well-identified moment, the quantitative literature typically estimates financing constraints off net book leverage, a moment used for instance by Hennessy and Whited (2007) and Midrigan and Xu (2014). In Section 5, we will discuss in more detail why estimations of the cost of financing constraints are sensitive to these assumptions when one targets net book leverage, and why targeting our preferred reduced-form moment β may yield more robust inference.

Fourth, we use data on equity issues to identify the cost of equity issuance (Hennessy and Whited, 2007). We compute the average ratio of net positive equity issuance to value-added – which is the relevant empirical counterpart for revenue $p_{it}q_{it}$ in the model. For each firm, we compute net equity issues as stock sales (COMPUSTAT item sstk) minus cash dividends (item dv) and share buybacks (item prstkc). We then take the maximum of this number and zero and normalize it by value-added. COMPUSTAT does not have a variable for value-added, so we use 60% of total sales (item sale). The implicit 40% gross margin ratio we are using is taken from Asker et al. (2014), Table C1. The targeted moment corresponds to the average of this ratio across all firms in our sample, .026.

3.4 Parameter Identification

This section discusses *local* identification of the parameters of the model: We show the relationship between empirical moments and model parameters around the main SMM estimate for (s, c, ρ, σ, e).

Appendix Figures C.1-C.5 offer visual evidence of how the targeted moments vary as a function of model parameters. To construct these figures, we first set all parameters (s, c, ρ, σ, e) at their estimated value, and then vary one of these parameters in partial equilibrium, i.e. holding fixed w and Q. Importantly, the comparative statics we report on these figures are *direct simulation output*: The relative smoothness of these plots gives us confidence in the robustness of our numerical procedure, which we attribute to the dense grid for capital (about 300 points), debt (29 points) and productivity (51 points), as well as to a large number of simulated observations (1,000,000 firms over 10 years). See Appendix A for details.

Figure C.1 shows that the reduced-form moment β, the impact of real estate collateral on investment, is non-monotonic in s, the collateral parameter. Intuitively, for lower values of s, firms' investment decisions are constrained by collateral availability, so that an increase in s allows firms to extract more debt and investment capacity out of a \$1 increase in collateral value, and β increases. For extreme values of s ($s \geq 1$), however, firms become unconstrained, and investment becomes independent of debt capacity so that β goes to 0. Around the SMM estimate (represented by a vertical line), the reduced-form moment β is a smooth and increasing function of s. Leverage is also smoothly increasing with the collateral parameter s. The first two panels of Figure C.1 also show that an increase in s leads to an increase in output volatility: When the firm becomes less constrained, its capital stock responds more to productivity shocks.

The adjustment cost parameter c is mostly identified by the autocorrelation of investment (Figure C.2). Large adjustment costs lead the firm to smooth investment over time, which leads to a large autocorrelation of investment. Larger adjustment costs to capital also lead to lower short-term output volatility. Similar to financing constraints, adjustment costs prevent firms from adjusting their capital stock in response to productivity shocks, making output less volatile. Figures C.3 and C.4 show that (1) the volatility of log-productivity, σ, has a nearly linear effect on output volatility at all horizons, while (2) the productivity persistence ρ mostly affects the long-term volatility of output. Combined together, these two observations are consistent with the idea that the ratio of the 1-year to 5-year output volatility allows us to identify the persistence parameter ρ. Note also that the persistence of productivity shocks has a sizable positive effect on the autocorrelation of investment. Firms can afford to delay their response to productivity shocks when these shocks are more persistent. Unsurprisingly, the cost of equity issuance e monotonically decreases with net equity issuance (Figure C.5).

Table 1 provides the elasticities of each moments with respect to estimated parameters – a simple transformation of the Jacobian matrix. Precisely, we compute for each moment m_n, and each parameter ω_k, the following elasticity (Hennessy and Whited (2007)):

$$\epsilon_{n,k} = \frac{\log m_n^+ - \log m_n^-}{\log \omega_k^+ - \log \omega_k^-} \approx \frac{\partial \log(\hat{m}_n)}{\partial \log(\hat{\omega}_k)},$$

where $\hat{\omega}_k$ is the parameter value at the SMM estimate and $\hat{m}_n$ the corresponding value for moment n. $\hat{\omega}_k^+$ (respectively $\hat{\omega}_k^-$) is the parameter value located right above (resp. below) on the grid used to plot Appendix Figures C.1-C.5. m_n^+ (resp. m_n^-) is the corresponding simulated moment obtained using parameter $\hat{\omega}_k^+$ (resp. $\hat{\omega}_k^-$), keeping the other parameters $\hat{\omega}_{k'}$ at their SMM estimate. Table 1 confirms the results in Appendix Figure C.1-C.5.

3.5 Estimation results

We report the results of the SMM estimation in Table 2. Each column corresponds to a model specification. Column (1) assumes no equity issuance ($e = +\infty$) and no adj costs ($c = 0$), column (2) allows for adjustment costs. Column (3) allows for both adjustment costs and equity issuance and constitutes our baseline specification. Column (4) of Table 2 contains the data.

Overall, these estimations show that the estimate of s, the pledgeability parameter, is quite robust to the introduction of adjustment costs and equity issuance. The estimate ranges from 0.20 to 0.25: each \$1 unit of capital provides about \$0.20 of debt capacity. This is reassuring, as it suggests that s is "pinned down" by the sensitivity of investment to real estate, and is not much affected by misspecification bias arising from the omission of real frictions or outside equity issuance. We explore misspecification bias more systematically in Section 5.

The estimated persistence and volatility of productivity are stable across specifications. The estimated persistence ranges from .85 in our baseline model to .92 in the model with no adjustment cost and no equity issuances. The estimated volatility is 13%. Relative to the literature, we estimate relatively small adjustment costs, $c = 0.004$. Intuitively, the model without adjustment costs can already generate a reasonably high autocorrelation of investment because of financing constraints. Note, however, that time-to-build in capital is already an important source of real friction in this model (Asker et al., 2014). Finally, we estimate a cost of equity issuance of \$0.09 per \$1 of new equity issued, which is in the ballpark of the existing empirical and structural literature (Hennessy and Whited, 2007).

Finally, Table 2 analyzes two non-targeted moments. We first look at the effect of real estate collateral on net debt changes, estimated by using $\Delta debt_{it}$ as a dependent variable in equation (1). Empirically, we estimate a significant coefficient of 0.06. All three specifications match this moment well (e.g., 0.070 in column (3)). We also consider the average net leverage ratio, which we define as net debt (dltt+dlc-che) divided by total assets (at). In our sample, the average net leverage is equal to 0.098. The estimated model in column (3) generates an average leverage ratio of 0.17, much larger than its empirical counterpart. We discuss leverage-based inference in greater detail in Section 5.

In the remainder of the paper, we use the model in column (3) with adjustment costs and equity issuance costs as our baseline specification.

3.6 Determinants of financing constraints

We briefly discuss how firm characteristics covary with financing constraints in the cross-section of simulated data. To identify financially constrained firms, we first simulate data using our baseline estimated model (Table 2, column (3)). For each simulated observation (i, t), we then compute its actual value and the value the firm would have if it started from the state variables, but financing frictions were removed forever. We label a firm as constrained when its constrained value is less than 95% of its unconstrained value.

Panel A in Figure 1 shows the share of financially constrained firms across 20 equal-sized bins of productivity. We find that less productive firms are on average more constrained. Because productivity is mean-reverting, low productivity firms typically experience positive productivity shocks but have a low level of capital, which prevents them from borrowing and investing. Similarly, growing firms are more likely to be financially constrained (Panel B). In Panel C, we report the share of financially constrained firms across 20 equal-sized bins of log-sales-to-capital ratio (log MRPK). Log-MRPK is closely related to distortions (e.g. Hsieh and Klenow (2009)). With Cobb-Douglas production and no friction, log-MRPK measures marginal revenue productivity and should be equated across firms. Our setting involves several frictions (time-to-build, adjustment costs, financing constraints) so that log-MRPK is not equal to the user cost of capital. Instead, Panel C shows that high MRPK firms tend to be more constrained on average since they have too little capital. The same intuition explains the result in Panel D, which shows that high Tobin's Q firms are more on average more likely to be financially constrained.

4 General Equilibrium Analysis

To quantify the aggregate effects of financing frictions, we now embed our estimated firm dynamics model in general equilibrium and simulate counterfactual economies.

4.1 General equilibrium model

By clearing the goods and labor markets, the model endogenizes aggregate demand Q and the real wage w introduced in the model of Section 2, equations (2)-(7).

Firms. A large number N of firms indexed by i produce intermediate inputs, in quantity q_{it}, at price p_{it}. Intermediates are combined into a CES-composite final good

$$Q_t = \left(\sum_{i=1}^{N} q_{it}^{\frac{\phi-1}{\phi}} \right)^{\frac{\phi}{\phi-1}}. \tag{8}$$

The final good is produced competitively. The demand for input i is thus given by $q_{it} = Q_t \left(\frac{p_{it}}{P_t} \right)^{-\phi}$, with $P_t = \left(\sum_i p_{it}^{1-\phi} \right)^{\frac{1}{1-\phi}}$. We normalize $P_t = 1$ and derive the demand function in equation (3).

Consumption and consumer behavior. The final good is used for (i) consumption, (ii) investment, and (iii) to pay for adjustment costs. The final good market equilibrium thus writes:

$$Q_t = C_t + \text{Adj. Cost}_t + I_t, \tag{9}$$

with C_t being aggregate consumption, Adj. $\text{Cost}_t = \sum_i \frac{c}{2} i_{it}^2 / k_{it}$ the sum of all adjustment costs, $I_t = \sum_i i_{it}$ aggregate investment, under our normalization $P_t = 1$.

A representative consumer maximizes utility over consumption and labor:

$$U_s = \sum_{t \geq s} \beta^{t-s} u_t \text{ with } u_t = C_t - \bar{L}^{-\frac{1}{\epsilon}} \frac{L_t^{1+\frac{1}{\epsilon}}}{1+\frac{1}{\epsilon}}, \tag{10}$$

with L_t aggregate hours worked, $\bar{L}$ a scaling constant, and ϵ the Frisch elasticity of labor supply. With quasi-linear preferences, the Hicksian, Marshallian and Frisch labor supply elasticities are all equal to ϵ. Labor supply is a static decision given by

$$L_t^s = \bar{L} w_t^{\epsilon}. \tag{11}$$

Steady state assumption and equilibrium definition. We assume that the economy is in steady-state. In steady-state, the consumption Euler equation ties the equilibrium interest rate r_t to the discount rate β, so the interest rate $r_t = 1/\beta - 1$ is pinned down throughout all counterfactuals. The "exogeneity" of r in this model is an outcome of our steady-state assumption. It holds for any additively separable utility function.

Intermediate good producers produce according to the Cobb-Douglas technology described in (2). The log productivity shocks z_{it} that they face have no aggregate component. Given our assumption that the number of firms is large, aggregate output Q and the wage w are constant over time. We are thus exactly in the case described in Section 2 and estimated in Section 3.

Given the normalization $P_t = 1$, the equilibrium (Q, w) of this economy is defined by two equations, the labor market equilibrium and the final good aggregator:

$$\bar{L} w^{\epsilon} = \sum_{i=1}^{N} l^d \left((Q, w) ; z_{it}, k_{it} (Q, w) \right) \tag{12}$$

$$PQ = \sum_{i=1}^{N} p_{it} q \left((Q, w) ; z_{it}, k_{it} (Q, w) \right). \tag{13}$$

$l^d(\cdot)$ is the numerically obtained labor demand function which is a function of each firm state variable and aggregate equilibrium (Q, w). Similarly $pq(\cdot)$ is the supply function, which, for each firm, associates state variables and macroeconomic conditions to its dollar sales. The equilibrium (Q, w) is the solution of these two conditions.

We solve this equilibrium using the following approach. We simulate data from our estimated model using arbitrary starting values for (Q_0, w_0). Following Sraer and Thesmar (2018), we then compute aggregate TFP and output using three sufficient statistics from the simulated data: The mean and the variance of log MRPK ($\log \frac{pq}{k}$ in the model), and the covariance of log-MRPK with log firm-level TFP. Sraer and Thesmar (2018) show that this approach yields aggregate output and TFP in one iteration for any starting point (Q_0, w_0). We then solve for aggregate wage w, employment L and total capital stock K.[10]

4.2 The aggregate effect of financing constraints

We now evaluate the aggregate effect of financing constraints based on our estimated model. Compared to the firm-level model, the macroeconomic model has a few additional parameters. Following Chetty (2012), we set the labor elasticity $\epsilon = 0.50$. We adjust $\bar{L}$ and the number of firms N so that the equilibrium parameter chosen for the estimation process ($Q = 1$ and $w = 0.03$) are actual equilibrium parameters when firm parameters are at the SMM estimate.

To measure the aggregate effect of financing constraints, we calculate aggregate TFP and output in log deviations from the "unconstrained" benchmark. The appropriate way to define the unconstrained benchmark in our model is to set equity issuance cost e to 0, rather than removing the collateral constraint. With $e = 0$, investment is unconstrained since equity is freely available to all firms. With no collateral constraint, firms would raise infinite debt for tax purposes. Our unconstrained benchmark thus corresponds to a model with free equity issuance and all other structural parameters – including the collateral constraint – unchanged. It also has the advantage of giving unconstrained firms the ability to benefit from the tax shield and lower their cost of capital, just like constrained ones. As we see below, as s increases, constrained and unconstrained economies behave more and more similarly.

Table 3, Panel A, Column (3) shows that, in our baseline model, financial frictions result in a large output loss of 7.1% relative to the unconstrained benchmark. The

[10]We use the following additional equations:

$$\log Y = \log TFP + \alpha \log K + (1 - \alpha) \log L$$
$$\log Y = \log w + \log L$$
$$\log L = \epsilon \log w$$

main channel for this output loss is the aggregate reduction in productive inputs: Relative to the unconstrained benchmark, employment is lower by 2.4%, and capital by 13.7%. Financing frictions also lead to input misallocation, although this misallocation channel is quantitatively less important: Aggregate TFP is lower by 1.4% in the estimated economy relative to the unconstrained benchmark. Together, these two channels reduce aggregate labor productivity, wages, and therefore labor supply, depressing employment. In this economy, however, the quantitatively important distortion induced by financing constraints is to prevent households from saving as much as they would want to (low capital stock), rather than allocating capital to the wrong firms (low TFP).

Table 3, Panel A also shows that adjustment costs tend to slightly attenuate the losses from financing constraints: the output loss is about 3 percentage points smaller in a model with adjustment costs (and no equity issuance, column (2)) than in a model with no adjustment cost and no equity issuance (column (1)); similarly, TFP losses are .4 percentage points smaller in the presence of adjustment costs. In the presence of adjustment costs, firms smooth out investment, which becomes less responsive to productivity shocks. As a result, financing constraints bind less often. As such, real and financial frictions do interact non-trivially in our model, although quantitatively, the role of this interaction is limited.

Figure 2 displays comparative statics for output and TFP losses with respect to the collateral parameter s. We fix all other parameters to their baseline estimates in column (3), Table 2. For 20 values of s around its estimated value, we solve the model and compute *aggregate TFP and output losses* from financing frictions. Figure 2 also reports the estimated collateral parameter s (vertical dark line), along with the 90% confidence band for this parameter (light blue bar). The precision of our estimate – a standard error of 0.045 for a point estimate of 0.25 – implies that for values of s in the 90% confidence interval, output losses are between 6.5 and 8%, and TFP losses between 1.1 and 1.5%.

Finally, to illustrate the role of general equilibrium effects in our quantitative exercise, we compute economic losses due to financing constraints *in partial equilibrium*. Precisely, we simulate a benchmark economy where firms face no equity issuance cost but assume that the aggregate demand shifter Q and the wage w remain constant equal to their value in the estimated economy. We then compare total output and efficiency in this benchmark economy relative to the estimated economy. As expected, output losses are much larger in partial equilibrium. When financial constraints are relaxed, employment responds significantly more in partial equilibrium as this exercise implicitly assumes that the labor supply is infinitely elastic, as opposed to the elasticity of .5 we use in general equilibrium. As a result, and given our Cobb-Douglas production function, the capital stock also increases more in partial equilibrium when financing frictions are relaxed. Note that in partial equilibrium, financing constraints artificially boost the measure of TFP: while financing frictions reduce the aggregate inputs used in the economy, the fact that the aggregate demand shifter is fixed implies that the economy with financing frictions is more efficient.

4.3 Productivity persistence and misallocation

The literature also discusses comparative static w.r.t. productivity persistence ρ. Recent papers emphasize that the persistence of productivity shocks should reduce distortions coming from financing frictions (Moll, 2014; Buera et al., 2011). Intuitively, if shocks are persistent enough, firms can accumulate internal funds to free themselves of financing constraints. Because the focus in this literature is about the misallocation of capital *across firms*, we compute here the cross-sectional s.d. of log MRPK ($\log p_i q_i / k_i$) as our measure of distortions. In our setting, this dispersion is directly linked with the log-TFP loss from all frictions, from financing constraints to adjustment costs and time-to-build in capital (Hsieh and Klenow (2009)).[11]

Figure 3 plots the dispersion of log-MRPK against ρ. We set parameters to their estimated value in Table 2, Column (3). We then vary ρ around its estimated value, and compute the equilibrium dispersion of log-MRPK (Hsieh and Klenow, 2009; Midrigan and Xu, 2014). To make the exercise meaningful, we maintain a constant level of unconditional productivity dispersion by setting $\sigma^* = \sigma^2/(1-\rho^2)$ constant, thus varying σ^2 accordingly (Moll (2014)). Figure 3 confirms that misallocation is significantly reduced when productivity shocks become more persistent. When ρ is set to .4, which is about one half of its estimated value, the dispersion of log-MRPK increases from .12 in our estimated model to about .25.

5 Robustness to misspecification

A key contribution of this paper is to ground the estimation of collateral constraints on a well-identified, reduced-form moment: The effect of shocks to collateral value on investment (β). The quantitative literature in structural corporate finance and macro-

[11] This is just a restatement of the result in Hsieh and Klenow (2009), combined with the fact that the distribution of log MRPKs in our simulated data is close to log-normal.

finance typically makes inference using a different moment, the leverage ratio. This section explores how estimations based on leverage vs. β differ in terms of robustness to misspecification.

5.1 Local Sensitivity to Leverage and β

Andrews et al. (2017) show that, as long as misspecification is local, misspecification bias for an estimate X is simply:

$$\hat{X} - X = \underbrace{(\nabla X)' \times [-(J'WJ)^{-1}J'W]}_{\text{sensitivity}}(m^* - m) \tag{14}$$

where $\hat{X}$ is the misspecified estimate and X its true value, W is the SMM weighting matrix, J the Jacobian matrix and ∇X the gradient of X with respect to structural parameters, all computed at the SMM estimate. $(m^* - m)$ is the difference between misspecified moments (m^*) and the true moments (m). Computing the sensitivity matrix is straightforward as ∇X, J and W are easy to calculate at the SMM estimate.

Following Andrews et al. (2017) prescription, we compute the sensitivity of aggregate TFP and output losses from financial constraints. Table 5, Panel A reports the (generalized) inverse of the Jacobian matrix. Panel B reports the gradient of output and TFP losses from financing constraints w.r.t. the parameters at their estimated value. Panel C constructs the product of the two, which corresponds to the sensitivity of output and TFP losses with respect to the estimated parameter. Table 5, panel C shows that the sensitivity of both TFP and output losses with respect to leverage is about twice larger than their sensitivity with respect to β. To illustrate this finding, consider a source of misspecification that leads to a mismeasurement of leverage of .1 and a mismeasurement of β of .1. Table 5, panel C implies that an estimation targeting leverage will result in a bias for TFP loss of .4 percentage points. In contrast, an estimation targeting β will result in a bias for TFP loss of -.2% only.

As emphasized in Andrews et al. (2017), such a finding is useful only if we can actually predict how alternative assumptions actually affect the true moments relative to the moments used in estimation (i.e., $m^* - m$). In other words, the above formula is only useful if we know how leverage and β would vary under particular sources of misspecification. We follow two approaches to evaluate the effect of misspecification on these two moments. In Section 5.2, we consider sources of misspecification that arise purely from *measurement issues*. We assume our model is correctly specified but the moments we use for estimation are mismeasured. In Section 5.3, we assume instead that the moments are correctly measured, but the model is misspecified. In both cases, we find that inference based on β is on average more robust to model misspecification that inference based on average leverage.

5.2 Moment misspecification

In this section, we consider sources of misspecification that arise purely from *measurement issues*. We assume that our model is correctly specified, but that the moments we target in estimation may be mismeasured. This approach is motivated by the vast corporate finance literature that has emphasized challenges to accurately measuring both the capital stock and debt. First, part of firms' actual capital is intangible and is, therefore, missing from our measure of total book assets (Peters and Taylor (2017)). Second, part of the capital stock may be in the form of operating leases and is therefore financed off-balance sheet (Rampini and Eisfeldt (2009), Li et al. (2016)). Third, firms can borrow from their suppliers through accounts payables to finance total assets (Barrot (2016)). Fourth, the net book value of PP&E may underestimate the true physical capital stock, as accounting depreciation is quite larger than the typical level of physical depreciation used in national accounts.

In Table 4, we adjust our measure of debt, tangible assets, investment, and total assets to account for each of these challenges. Appendix section D explains in detail the adjustments we do to obtain these measures. We then calculate average leverage in our sample and estimate β using these alternative definitions. Table 4 shows that average leverage is quite dispersed across these six alternatives: the baseline leverage is 9.3%; when we account for intangible assets, leverage decreases to 7.2%; when we include leases in both debt and assets, leverage goes up to 20.2%. Across these alternatives, the standard deviation of the difference between our baseline leverage and the modified leverage is .058. At the same time, Table 4 also shows that β is more stable across these alternative specifications. Our baseline estimate for β is .06; when we include current assets in capital, β goes down to .028; when we include intangible capital in our measure of capital, β goes up to .083.[12] Across these alternatives, the standard deviation of the difference between our baseline β and the adjusted β is .021, about three times smaller than for average leverage.

Taken together, these results suggest a lower scope for misspecification when estimating TFP or output losses by targeting β: (1) the estimates for TFP and output

[12] Note that β remains statistically significant at the 1% confidence level across all these specifications.

losses are twice more sensitive to leverage than to β (2) an exploration of misspecifications due to mismeasurement of debt and capital reveals that average leverage in these alternative specifications varies significantly more (relative to its baseline value) than β.

5.3 Model misspecification

The previous section considered misspecifications arising purely from measurement issues. In this section, we consider a different type of misspecification: we assume that the moments are correctly measured, but that the model is misspecified. We consider several dimensions of model misspecification and investigate the robustness of inference based on β relative to inference based on leverage. Because we are interested in potentially large deviations from our baseline model, we depart from the local approximation approach of Andrews et al. (2017) and instead rely on a Monte-Carlo approach.

Concretely, we proceed in three steps: (1) we simulate data under an auxiliary model, (2) we use simulated data to estimate our baseline (hence misspecified) model, targeting either of the two moments and (3) we compare the true TFP or output loss with the two misspecified estimates. Since this approach requires to "specify the misspecification", our conclusions only apply to the particular set of alternative models we consider. We overcome this limitation by exploring 4,000 auxiliary models.

5.3.1 Auxiliary models

We describe here the class of auxiliary models we use to simulate data. We expand the baseline model in 6 directions, each described by one additional parameter of varying intensity. In what follows and for clarity, the 6 added parameters are in **bold**.

With the first group of three parameters, we investigate misspecifications that relate to how we model the capital stock. First, we include intangible capital in the model. We assume that intangible capital (1) is a perfect complement to physical capital (i.e. the production function is Leontieff in intangible and tangible capital) (2) cannot be collateralized (3) is unobserved by the econometrician and (4) depreciates at the same rate as tangible capital. We parametrize this modification of the baseline by $\boldsymbol{\mathcal{I}}$, the fraction of intangible capital, which we allow to go from 0 to 40% (Peters and Taylor, 2017). As a result of these assumptions, the only modification to the model is that cash-flows from investment are now given by:

$$\frac{1}{1-\boldsymbol{\mathcal{I}}}(-k_{it+1}+(1-\delta)k_{it}+\tau\delta k_{it}).$$

Our second specification considers the case where *tangible* capital is itself mismeasured by a factor $\boldsymbol{U}$. While in the model the tangible capital stock is k_{it}, the econometrician only observes $(1-\boldsymbol{U})k_{it}$. This factor is meant to account for operating leases, which represents productive capital that does not appear on the balance sheet. It also potentially accounts for the sizable discrepancy between accounting depreciation (about 15% in our data) and physical obsolescence of around 5-6% in the macro-finance literature (e.g. Midrigan and Xu (2014)). We restrict $\boldsymbol{U} \in [0, 0.33]$, so that the econometrician may miss up to one-third of the true tangible capital stock. In our last specification, we allow the price of real estate p_t to be mismeasured. We assume that the log measurement error follows the same AR1 process as actual log prices but with an innovation volatility $\boldsymbol{\sigma_u} \in [0; .03]$ equal to up to half of the actual innovation volatility of log prices (which is 0.06). This captures the fact that our price data are not granular enough to capture the real variation of each firm's real estate prices. This will affect inference using our moment, but not inference using leverage.

In the second group of 3 parameters, we allow for unmeasured sources of debt capacity / borrowing needs. First, we note that our model may be misspecified in that some of the debt capacity may be unsecured and therefore less related to the amount of capital stock available. We give firms a fixed amount of debt capacity $\boldsymbol{d_0}$, so that their debt constraint now writes:

$$(1+r)d_{it+1} \leq \boldsymbol{d_0}\bar{k} + s\left((1-\delta)k_{it+1} + \mathbb{E}(p_{t+1}|p_t) \times h\right)$$

where $\bar{k}$ is a scaling factor equal to the mean capital stock in the sample. We let $\boldsymbol{d_0} \in [0, 0.4]$. Second, we model the fact that operating leases offer an additional source of secured debt, which does not appear on the firm's balance sheet. We assume that observed net leverage is given by:

$$lev_{it} = \frac{d_{it} + \boldsymbol{d_0}\bar{k}}{k_{it}} - \boldsymbol{\kappa}$$

where $\boldsymbol{\kappa}$ captures the amount of debt capacity that the econometrician does not see when computing the classical net book leverage ratio. We allow $\boldsymbol{\kappa} \in [-0.3, 0.3]$ to be negative in order to capture the notion that some of the existing debt capacity may be committed to non-investment use (i.e. working capital finance). Third, we allow for mismeasurement in the effective tax rate that firms are facing. While the econometrician assumes $\tau = .33$, we assume that the true data-generating tax rate $\boldsymbol{\tau}$ may deviate from the baseline. We cover $\boldsymbol{\tau} \in [0.2, 0.4]$ in order to allow firms to face a lower effective tax rate than the statutory one, for instance, because of tax credits or tax loss carry-forwards. This affects the attractiveness of debt and therefore capital structure.

We end up with an alternative data-generating process that, in addition to our baseline model, accounts for intangible capital ($\boldsymbol{\mathcal{I}}$), tangible capital mismeasurement (**U**), measurement error of real estate value ($\boldsymbol{\sigma_u}$), unobserved debt capacity ($\boldsymbol{\kappa}$), unsecured debt capacity ($\boldsymbol{d_0}$) and error in tax rate ($\boldsymbol{\tau}$). We now describe how we estimate misspecification biases in a cross-section of 4,000 such simulated economies.

5.3.2 Monte-Carlo approach

In this section, we assess the exposure of inference based on β and inference based on leverage to misspecification bias. To do so, we start by constructing a large cross-section of hypothetical economies generated under alternative models. Each of these alternative data-generating models is characterized by the 5 baseline structural parameters (ρ, σ, s, c, e) and 6 "misspecification" parameters $\boldsymbol{\Theta} = (\boldsymbol{\mathcal{I}}, \boldsymbol{U}, \boldsymbol{\sigma_u}, \boldsymbol{d_0}, \boldsymbol{\kappa}, \boldsymbol{\tau})$. We draw 4,000 such sets of 11 parameters uniformly. For each draw of these parameters, we follow the steps below:

1. We solve the *true* model, simulate a dataset, and calculate the log TFP and output losses from financing constraints under this true model. We do this by comparing the TFP/output of the true model, with the same aggregates under a version of this model with no equity issuance cost (setting $e = 0$).

2. Using the simulated data, we calculate all the moments used in the estimation. We then use these moments to estimate the 5 parameters (ρ, σ, s, c, e) of our baseline model which assumes $\boldsymbol{\Theta} = (0, 0, 0, 0, 0, 0.33)$ and it is therefore *misspecified*. We perform two estimations:

   (a) We target the same moments as in Table 2, column 3, which include, in particular, the reduced-form moment β. We then compute log TFP and output losses from financing constraints given the estimated parameters.

   (b) We target the same moments but replace the reduced-form moment β with average leverage. We again compute log TFP and output losses of financing constraints given these alternative estimated parameters.

3. For each of these estimated TFP and output losses, we calculate the misspecification error, i.e. the difference between estimated TFP/output loss and the true TFP/output loss, which we can compute since we control the true data-generating process.

We obtain 4,000 misspecification biases for TFP losses and output loss when we target leverage and 4,000 misspecification biases for TFP losses and output loss when we target β instead. We normalize each one of these misspecification biases by their averages across all simulations to use a common scale (average TFP/output loss is 1.1%/6.4% in the simulations). We describe the entire procedure in more detail in Appendix B. In particular, we face a numerical challenge. The estimation of one model takes approximately one day so that we cannot solve 8,000 models in a reasonable time. Instead, we resort to a polynomial interpolation that we describe in Appendix B.

5.3.3 Results

Figure 4 reports the distribution of misspecification errors for output loss (Panel A) and TFP loss (Panel B) across all possible alternative models. While misspecification errors can be sizable under both approaches, they are on average close to zero when we target the reduced-form moment β. When targeting leverage, however, the distribution of errors is more dispersed and significantly biased downward: the average misspecification bias for both output and TFP loss is about 30% of the average true loss across all models (remember that both these errors for TFP and output loss are both rescaled by their average across simulations).

To assess how misspecification errors depend on the misspecified parameters, we report in Table 6, the results of the following regression across simulated economies i:

$$\frac{\hat{X}_i - X_i}{\frac{1}{N}\sum_j X_j} = a + b\frac{\boldsymbol{\mathcal{I}}_i}{\max_j \boldsymbol{\mathcal{I}}_j} + c\frac{\boldsymbol{U}_i}{\max_j \boldsymbol{U}_j} + d\frac{\boldsymbol{\sigma}_{\boldsymbol{u},i}}{\max_j \boldsymbol{\sigma}_{\boldsymbol{u},j}} + e\frac{\boldsymbol{d}_{\boldsymbol{0},i}}{\max_j \boldsymbol{d}_{\boldsymbol{0},j}} + f\frac{\boldsymbol{\kappa}_i}{\max_j \boldsymbol{\kappa}_j} + g\frac{\boldsymbol{\tau}_i - 0.33}{\max_j (\boldsymbol{\tau}_j - 0.33)} + \epsilon_i$$

where $\frac{\hat{X}_i - X_i}{\frac{1}{N}\sum_j X_j}$ is the misspecification error for statistics X scaled by its average across simulations. There are 4 sets of results, depending on whether we are targeting β or leverage, and whether we are calculating TFP or output loss. We do not report t-statistics since this is a simulated sample – all coefficients will end up significant with enough simulations. We do show, however, in Appendix Figures C.6-C.7, that the number of simulations is large enough to ensure smooth dependence of misspecification errors as a function of the parameters governing the misspecification (Θ) .

Results from Table 6 show that, for these particular sources of misspecification, inference based on β typically yields a more robust estimation than an inference based on leverage. First, note that the constant is close to zero in both types of estimation. This is consistent with the idea that, if the model is correctly specified, the inference is correct under both reduced-form moment β- and leverage-targeting.

Second, the leverage-based inference is sensitive to the presence of misspecifications on debt – a problem our reduced-form moment β does not have. The slope of misspecification errors with respect to misspecification arising from unsecured debt (resp. unobserved debt capacity) is about four times larger (resp. 40 times larger) when the model is estimated by targeting leverage than by targeting β. These results are intuitive: the estimation of β does not require any information on debt from firms' balance sheet. The signs of the slopes in Table 6 are also intuitive. A higher share of unobserved debt capacity implies that observed leverage is too low relative to firms' true leverage. An estimation targeting leverage will thus underestimate true debt capacity, leading to an overestimation of losses generated by financing constraints (1.2 for both TFP and output losses). Again, since β is estimated without information on debt, the estimation based on β will not suffer from this issue.

Third, the leverage-based inference is also quite sensitive to sources of misspecification related to the capital stock. If the share of unobserved intangible assets increases by 1 percentage points, the average misspecification error for TFP loss goes down by 41 basis points if the model is estimated by targeting leverage; in contrast, if the model is estimated by targeting β, the average misspecification error for TFP loss only goes down by .5 basis points, i.e. 20 times less. The fact that the slope is negative is intuitive. Leverage-based inference overestimates the true leverage ratio as the true capital stock is larger than the measured assets. As a result, estimations targeting leverage underestimate the true extent of financing constraints. Similar conclusions hold qualitatively for other sources of misspecification, such as the existence of physical capital that is not measured in PP&E.

Last, we investigate two additional sources of misspecification: Measurement error in real estate prices and in the effective corporate income tax rate. Intuitively, measurement error in real estate prices only affects inference based on β, but not inference based on leverage. The misspecification bias for β-based estimates is, however, modest in size. If half of the innovation in real estate prices was pure noise, log

TFP losses would be overestimated by about 13% of the average TFP loss across simulations. The reason for this upward bias is intuitive: noise in real estate prices creates a downward bias in the investment regression, which our estimation wrongly attributes to a smaller collateral parameter s, leading to stronger financing constraints.

Misspecification related to firms' tax rate leads to similar biases for estimates based on β and estimates based on leverage. In both cases, if taxes are higher than their baseline value (33%), firms are more likely to sacrifice debt capacity to enjoy more tax shield. They are thus more constrained and removing constraints increases TFP and output more than under the baseline tax rate. This effect is independent of which moment is targeted in the estimation. In particular, inference based on β cannot address this issue.

Conclusion

This paper provides a quantification of the aggregate effects of a specific source of financing frictions, collateral constraints. We build a standard dynamic general equilibrium model with heterogeneous firms and collateral constraints. Our methodological contribution is to estimate this model by targeting a reduced-form moment, the observed sensitivity of investment to exogenous shocks to collateral values. The model suggests that financing constraints generate an output loss that is as high as 7%. This output loss can be broken down into three channels: (1) 1.4% TFP loss (misallocation) (2) 13.7% lower capital stock (3) 2.4% employment loss.

Our paper builds a bridge between the reduced-form corporate finance literature, which aims to provide well-identified evidence on credit frictions and the quantitative macro-finance literature. Our approach stands in contrast with the common practice in the quantitative literature, which mostly bases the inference of financial frictions on firms' average leverage ratio. Beyond the intuitive appeal of matching a well-identified moment specifically identifying financial frictions, our paper offers several analyses to show the robustness of this inference to misspecification bias. We consider the local approach of Andrews et al. (2017) and show that estimated output and TFP losses are less sensitive to β than to leverage. We also perform thousands of Monte-Carlo simulations to show that inference based on β is on average more robust to certain types of model misspecification that inference based on average leverage. Beyond the particular moments used in this paper, we believe that the combination of well-identified moments emphasized in the reduced-form literature and structural modeling is an extremely fruitful avenue for finance research. We hope to pursue this agenda further in future research.

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Tables

Table 1: **Elasticity of Moments with respect to Parameters**

	s.d. $\Delta \log$ sales	s.d. $\Delta_5 \log$ sales	Net debt / assets	$\beta(Inv, RE)$	Autocorr. Invest.	Equity issues / value added
Pledgeability s	.066	.071	1.1	1.2	-.64	-.23
Adjustment cost c	-.02	-.013	.029	-.0058	.31	-.071
Volatility σ	1	1.1	-.7	.7	.26	3.8
Persistence ρ	.81	2.1	-.76	-2.5	5.5	13
Issuance cost e	-.057	-.13	-.2	.21	-.72	-2.1

Note: This table reports the elasticity of simulated moments with respect to the estimated structural parameters. First, we start with the SMM estimate $\widehat{\boldsymbol{\Omega}}$ of the parameters $\boldsymbol{\Omega}$. For each $k = 1, \cdots, 4$, we set $\omega_l = \hat{\omega}_l$ for all $l \neq k$, and vary the parameter ω_k around the estimated $\hat{\omega}_k$ in order to compute the elasticity of moments to parameters in the vicinity of the SMM estimate. For each moment m_n, we compute

$$\epsilon_{n,k} = \frac{\log m_n^+ - \log m_n^-}{\log \omega_k^+ - \log \omega_k^-} \approx \frac{\partial \log(\hat{m}_n)}{\partial \log(\hat{\omega}_k)},$$

where $\hat{m}_n$ is the n^{th} data moment. m_n^+ is the moment based on data simulated with parameter $\hat{\omega}_k^+$. Likewise, m_n^- is the average of moments based on data simulated with parameters ω_k^-. ω_k^+ and ω_k^- are parameter values right above and right below the SMM estimate $\hat{\omega}_k$, when the interval of definition of ω is graded on a scale going from 0 to 10 as in Figures C.1-C.5. *Reading*: Around the SMM estimate, a 1% increase in s is associated with a 1.2% decrease in the sensitivity of investment to real estate and a 1.1% increase in leverage.

Table 2: **Parameter Estimates (SMM)**

Specification:	(1) Model 1: $c = 0, e = +\infty$	(2) Model 2: $c > 0, e = +\infty$	(3) Model 3: $c > 0, e > 0$	(4) Data
Panel A: Estimated Parameters				
ρ	0.922	0.893	0.851	
	(.013)	(.020)	(.017)	
σ	0.124	0.134	0.131	
	(.007)	(.004)	(.003)	
s	0.196	0.216	0.250	
	(.078)	(.080)	(.048)	
c	0	0.008	0.004	
		(.003)	(.002)	
e	$+\infty$	$+\infty$	0.091	
			(.012)	
*Panel B: Moments (targeted in **bold**)*				
SD 1-year sales growth	**0.327**	**0.327**	**0.327**	0.327
SD 5-year sales growth	**0.913**	**0.912**	**0.912**	0.912
Real-Estate to assets	**0.140**	**0.140**	**0.141**	0.140
$\beta(Inv, RE)$	**0.060**	**0.060**	**0.060**	0.060
Autocorrelation of Inv	0.041	**0.165**	**0.165**	0.165
Net Equity Issuance to value added	0	0	**0.026**	0.026
$\beta(D, RE)$	0.059	0.053	0.070	0.060
Net debt to assets	0.030	0.090	0.171	0.098

Note: This table reports the results of our SMM estimations. The estimation procedure is described in the text and in Appendix A. Columns (1)-(3) correspond to SMMs using different models. Column (1) assumes no adjustment cost and infinite cost of equity issuance ($c = 0, e = +\infty$). Column (2) introduces adjustment costs but maintains $e = +\infty$. Column (3) further allows for finite cost of equity issuance. For each of these estimations, Panel A shows the estimated parameters, along with standard errors (obtained via bootstrapping) in parenthesis. Panel B shows the value of a set of moments, measured on simulated data (with 1,000,000 observations). Moments in bold are the ones that are targeted in the estimation. The other moments are not targeted. The last column (labeled "data") reports the empirical moments.

Table 3: **Aggregate Effects of Collateral Constraints**

	(1)	(2)	(3)
Specification:	Model 1	Model 2	Model 3
	$c = 0, e = +\infty$	$c > 0, e = +\infty$	$c > 0, e > 0$
Panel A: *General equilibrium results*			
Δ log(TFP)	0.031	0.027	0.014
Δ log(Output)	0.151	0.120	0.071
Δ log(wage)	0.101	0.080	0.048
Δ log(L)	0.051	0.040	0.024
Δ log(K)	0.282	0.215	0.137
Panel B: *Partial equilibrium results (holding* (Q, w) *fixed)*			
Δ log(TFP)	-0.040	-0.029	-0.020
Δ log(Output)	0.400	0.320	0.189
Δ log(wage)	-	-	-
Δ log(L)	0.400	0.320	0.189
Δ log(K)	0.531	0.417	0.254

Note: This table reports the results of the counterfactual analysis for different SMM parameter estimates. The general equilibrium analysis is described in Section 4 and reported in Panel A. Columns (1)-(3) correspond to the three different models described in Columns (1)-(3) of Table 2: Column (1) assumes no adjustment cost ($c = 0$) and infinite cost of equity issuance ($e = +\infty$). Column (2) allows for adjustment cost but still assumes infinite cost of equity issuance. Column (3) also allows for finite cost of equity issues. Panel B implements the same methodology, except that it holds the aggregate demand shifter Q and wage w constant. Results in both panels are shown as log deviations from the constrained estimated model to the unconstrained benchmark. The unconstrained benchmark correspond to an equilibrium where firms face the same set of parameters as in the SMM estimate – reported in the same column, Table 2, panel A – but do not face a constraint on equity issuance ($e = 0$). In this unconstrained benchmark, investment reaches first best, but firms still benefit from the debt tax shield. *Reading*: In column 1 (no adjustment cost, no equity issuance), the aggregate TFP loss compared to a benchmark without financing constraints is 3.1%.

Table 4: **Average Leverage Ratios and β Using Alternative Definition**

	Definition	D	Assets	K	I	Leverage =D/Assets	β
1	Standard	dltt+dlc-che	at	ppent	capx	0.093	0.060
2	Intangible	dltt+dlc-che	at+$k_{\text{int}}^{\text{off-bs}}$	ppent+k_{int}	capx +xrd+.3×xsga	0.072	0.083
3	Leasing 1	dltt+dlc-che +lease	at+lease	ppent+lease +lease-lease(t-1)	capx	0.202	0.065
4	Leasing 2	dltt+dlc-che +PV(lease)	at	ppent	capx	0.130	0.060
5	Account payables	dltt+dlc-che +ap	at	ppent+act	capx+act-act(t-1)	0.201	0.028
6	Real depreciation	dltt+dlc-che	at+K-ppent	K	capx	0.074	0.070
7	All adjustments	dltt+dlc-che +lease+ap	at+K-ppent +lease+$k_{\text{int}}^{\text{off-bs}}$	ppent+k_{int} +lease+act	capx+act-act(t-1) +lease-lease(t-1) +(1-τ)(xrd+.3×xsga)	0.184	0.037

Source: COMPUSTAT. The sample corresponds to the sample of firms in Chaney et al. (2012). We calculate the average leverage ratio and estimate β under specific sources of misspecification. We use the following COMPUSTAT item: at is total assets; dltt is total long-term debt; dlc is debt in current liability; che is cash and short-term investment; ppent is property, plant and equipment; capx is capital expenditures; xrd is R&D expense; xsga is SG&A; act is total current assets; ap is account payables. k_{int} is intangible capital, and $k_{\text{int}}^{\text{off-bs}}$ its off-balance sheet part, from Peters and Taylor (2017). lease corresponds to leased operating capital and is calculated following an approach similar to Rampini and Eisfeldt (2009). For each firm-year, we compute l_{it}, the ratio of lagged 1-year rental commitments (mrc1) to the rental cost of assets, which we measure as depreciation (dp) plus 10% of total assets (at). We trim observations for which this ratio is above 1 or below 0, and set it to 0 when mrc1 is missing. We then multiply this ratio by total assets (at) to estimate the value of operating capital and implicit debt, assuming a leverage of one on operating capital. To calculate PV(lease), we start from the next five years commitments (mtr1-5), spread expected commitment (mrtca) over these five years equally, and calculate the present value of these commitments at a 10% discount rate. K corresponds to the capital stock calculated using a perpetual inventory method. For each firm, we take PP&E (ppent) in the first fiscal year post-1981, depreciate it every year at 6% as in Midrigan and Xu (2014) and increase it with capital expenditures (capx) and decrease it with sales of property (sppe).

Table 5: **Calculating the sensitivity of Baseline Estimates to Moments**

Panel A: Sensitivity Matrix $\Lambda = -(J'WJ)^{-1}J'W$

	SD 1-year sales growth	SD 5-year sales growth	Real-Estate to assets	Net Debt to assets	β (Inv, RE)	Autocorr. of inv.	Net equity iss. to value added
ρ	2.54	-0.89	-0.08	0.24	-0.43	0.09	0.59
σ	-0.74	0.11	0.01	0.01	0.03	-0.07	0.11
s	0.19	-0.27	0.01	-1.04	-0.37	0.09	0.98
h	0.32	-0.29	-1.44	0.11	-0.08	-0.06	0.67
c	-0.14	0.03	0.01	-0.04	0.02	-0.07	0.14
e	0.75	-0.40	-0.02	0.22	-0.20	0.04	1.73

Panel B: Gradients of log output and TFP losses

	ρ	σ	s	h	c	e
($\nabla \log$ Output loss)'	0.24	0.76	-0.07	0.00	-1.48	0.36
($\nabla \log$ TFP loss)'	0.02	0.12	-0.02	0.01	-0.03	0.10

Panel C: Gradient-adjusted sensitivity Matrix: $\nabla' \times \Lambda$

	SD 1-year sales growth	SD 5-year sales growth	Real-Estate to assets	Net Debt to assets	β (Inv, RE)	Autocorr. of inv.	Net equity iss. to value added
$(\nabla \log$ Output loss$)' \times \Lambda$	0.51	-0.29	-0.03	0.27	-0.15	0.09	0.57
$(\nabla \log$ TFP loss$)' \times \Lambda$	0.02	-0.04	-0.01	0.04	-0.02	0.00	0.17

Note: This table reports the derivative of estimates of TFP loss and output loss with respect to moments, computed at the SMM estimate. This calculation follows Andrews et al. (2017). Panel A reports the sensitivity matrix, which is defined as $\Lambda = -(J'WJ)^{-1}J'W$ where J is the Jacobian matrix estimated at the SMM estimate. Panel B reports ∇ the gradient of log TFP and output losses with respect to parameters at the SMM estimate. Panel C reports the sensitivity of these aggregate losses w.r.t. moments, $\nabla' \times \Lambda$.

Table 6: Estimation Error and Distance from Correct Specification

Relative error in estimation of:	$\log$ TFP loss		$\log$ Output loss	
Misspecified SMM targets:	β	Leverage	β	Leverage
	(1)	(2)	(3)	(4)
Misspecification parameters:				
Intangible capital share ($\boldsymbol{I}$)	-.0056	-.41	-.0021	-.39
Unobserved physical capital share ($\boldsymbol{U}$)	-.19	-.34	-.18	-.33
Price measurement error ($\boldsymbol{\sigma_u}$)	.12	-.0033	.11	-.0058
Unobserved debt capacity - need ($\boldsymbol{d_0}$)	.028	1.2	.041	1.2
Fixed unsecured debt ($\boldsymbol{\kappa}$)	.098	-.43	.075	-.42
Actual tax rate - 33% ($\boldsymbol{\tau} - 0.33$)	-.73	-.54	-.68	-.49
Constant	.063	.14	.065	.13
Observations	4,000	4,000	4,000	4,000
R^2	0.32	0.74	0.29	0.73

Note: We simulate datasets from 4,000 alternative models. Each alternative model correspond to the baseline model augmented in six different dimensions described in Section 5.3.3. Six "misspecification" parameters control the degree of departure from the baseline model along these dimensions: $\boldsymbol{\Theta} = (\boldsymbol{\mathcal{I}}, \boldsymbol{U}, \boldsymbol{\sigma_u}, \boldsymbol{d_0}, \boldsymbol{\kappa}, \boldsymbol{\tau})$. We estimate the baseline (misspecified) model on these 4,000 datasets using two separate approaches: one estimation targets leverage; another targets the reduced-form moment β. We then regress:

$$\frac{\hat{X}_i - X_i}{\frac{1}{N}\sum_j X_j} = a + b\frac{\boldsymbol{\mathcal{I}}_i}{\max_j \boldsymbol{\mathcal{I}}_j} + c\frac{\boldsymbol{U}_i}{\max_j \boldsymbol{U}_j} + d\frac{\boldsymbol{\sigma}_{\boldsymbol{u},i}}{\max_j \boldsymbol{\sigma}_{\boldsymbol{u},j}} + e\frac{\boldsymbol{d}_{\boldsymbol{0},i}}{\max_j \boldsymbol{d}_{\boldsymbol{0},j}} + f\frac{\boldsymbol{\kappa}_i}{\max_j \boldsymbol{\kappa}_j} + g\frac{\boldsymbol{\tau}_i - 0.33}{\max_j (\boldsymbol{\tau}_j - 0.33)} + \epsilon_i$$

where X stands for the estimated TFP/output losses and i index alternative models. Standard errors are omitted because they are irrelevant in this cross-section of simulations, but the number is large enough to ensure smooth, linear, relationships as shown in Appendix Figures C.6 and C.7. *Reading*: When the fraction of intangible capital increases from 0 to .5 (maximum misspecification), the misspecification bias on TFP losses estimated by targeting leverage increases from zero (correctly specified) to 41% of the average TFP loss in the cross-section.

Figures

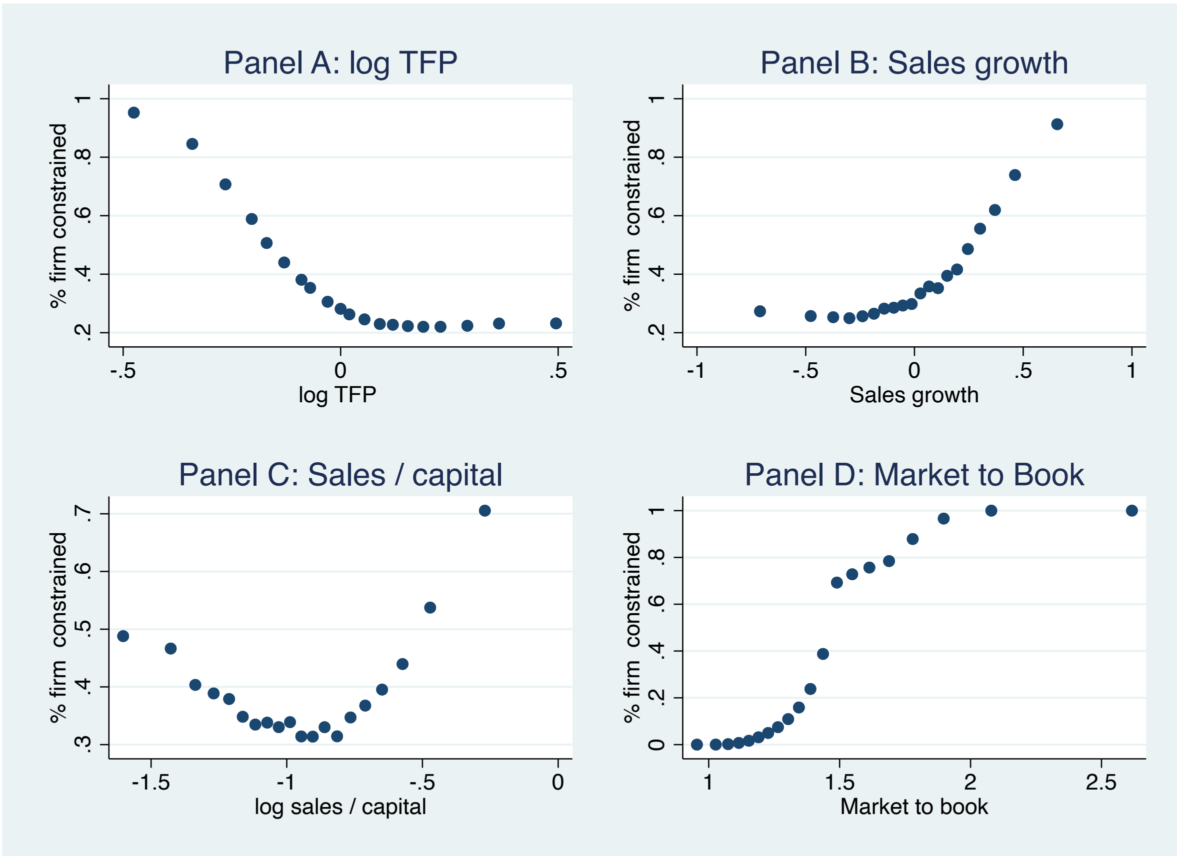


Figure 1: **Financing Constraints as a Function of Firm Characteristics**
Note: This figure shows how the extent of financing constraints covaries with firm characteristics, in the cross-section of simulated firms. We simulate a dataset of 1,000,000 firms over 215 years using parameters from our preferred specification (Table 2, Panel A, column 3). We remove the first 200 years to make sure firms are in steady-state. For each characteristic x, we then sort firms into 20 equal-sized bins of x, and, for each bin, compute the average share of constrained firms. We label a firm-year "constrained" if its market value is less than 95% of its unconstrained market value. Unconstrained market value is computed using the same set of state variables (z, k, b) at the beginning of the period, and a model for which the cost of equity issuance e is set to 0. We use the following conditioning variables x: z (Panel A), $\log pq_t - \log pq_{t-1}$ (Panel B), $\log \frac{pq}{k}$ (Panel C), and $\frac{V}{k}$ (Panel D).

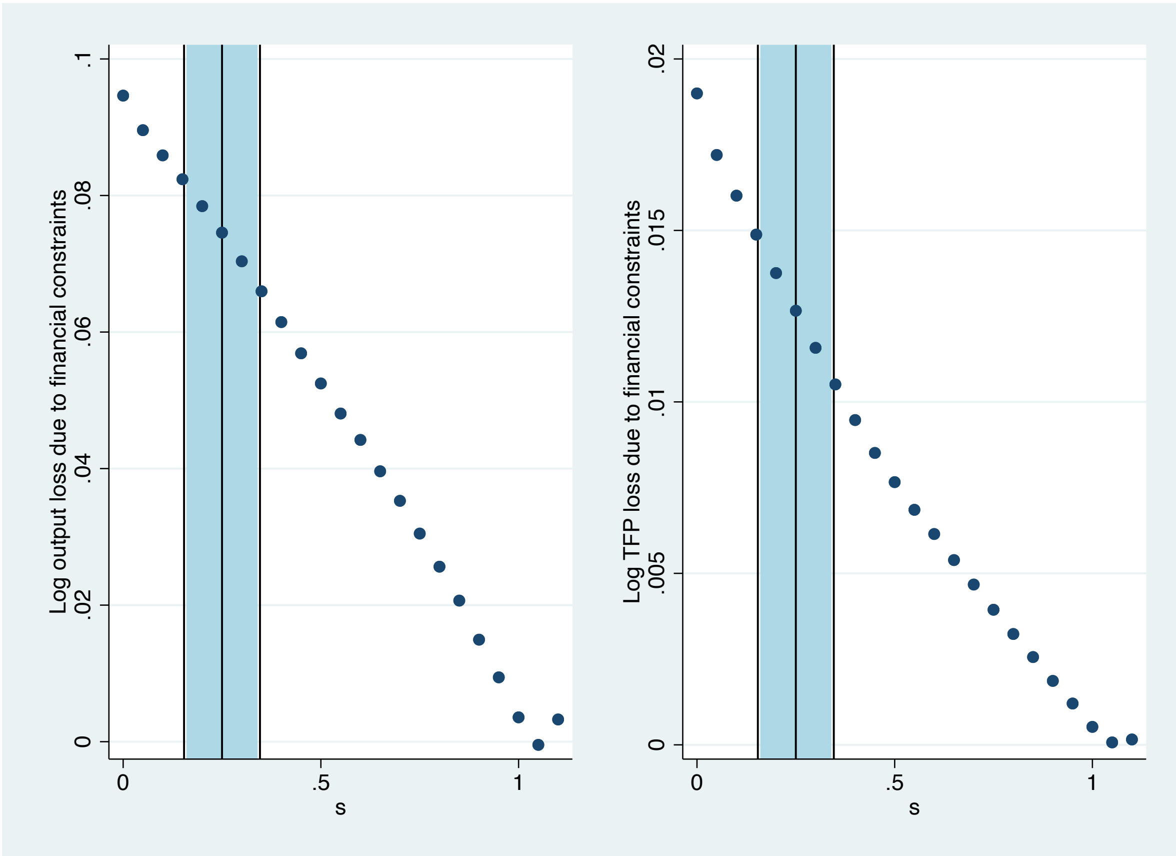


Figure 2: **General equilibrium effect of pledgeability** s

Note: This figure reports the general equilibrium effect of changing the collateral parameter s from 0 (the capital stock cannot be pledged as collateral) to 1 (100% of the capital stock can be pledged to lenders). For each value of s, we first compute aggregate output and TFP in a general equilibrium economy where firms face a collateral parameter s and all other parameters are set to their estimate in Table 2, Panel A, column 3. We then compute aggregate output and TFP in another general equilibrium economy where firms face the same collateral parameter s and all other parameters are set to their estimate in Table 2, Panel A, column 3, except for the equity issuance parameter which is now set to 0. This other economy corresponds to the unconstrained benchmark: in the absence of equity issuance costs, firms' investment will be first-best. For each value of s, we then compute the log difference of output and TFP between these two economies. The vertical black line corresponds to the SMM estimate of s (0.25) and the shaded blue bar its 90% confidence interval. Reading: When s increases from .1 to .6, the output loss relative to the unconstrained benchmark goes from 10% to 5%.

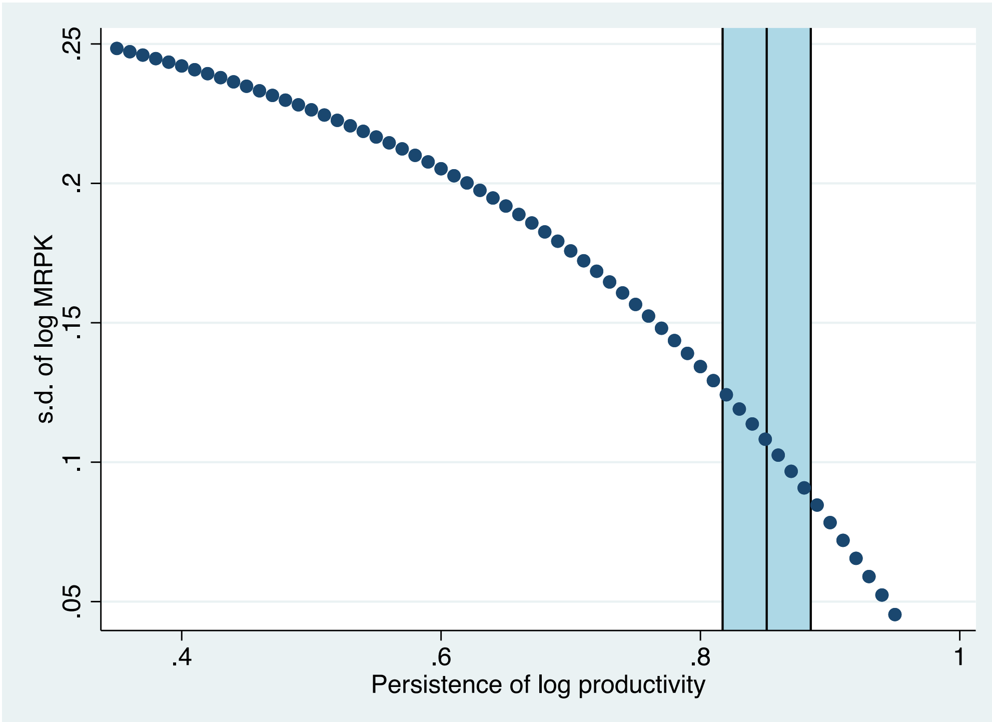
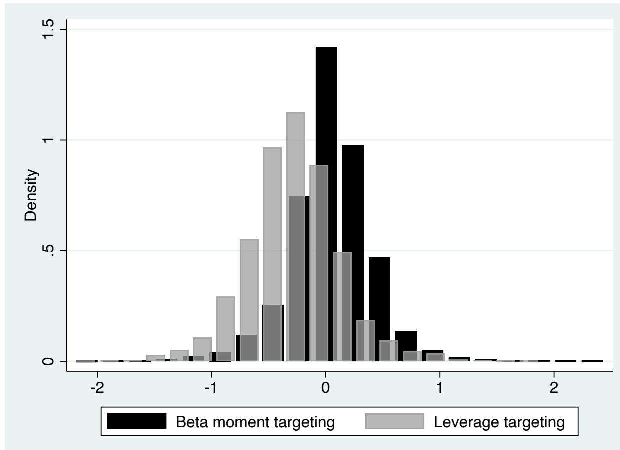


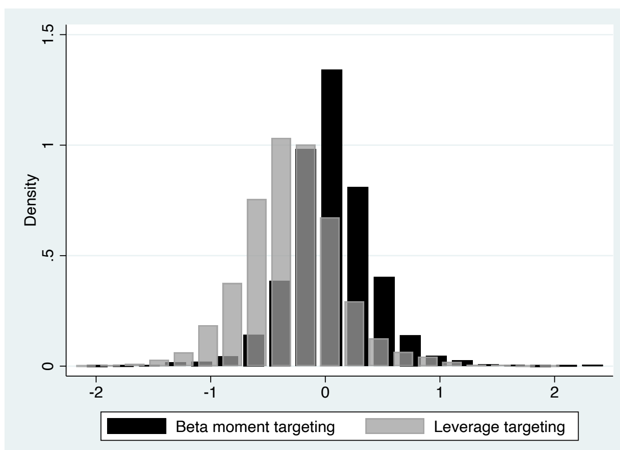
Figure 3: **Productivity Persistence ρ and the Dispersion of log-MRPK**
Note: This figure reports the effect of changing the log productivity persistence ρ from .35 (low persistence) to .95 (high persistence) on capital misallocation. Following Hsieh and Klenow (2009), we measure misallocation as the s.d. of $\log$ output to capital ratio (log-MRPK=$\log(\frac{py}{k})$). For each value of ρ, we compute the dispersion in log-MRPPK in a general equilibrium economy where the productivity persistence is ρ and all other parameters are set to their estimate in Table 2, Panel A, column 3. The vertical black line correspond to the SMM estimate of ρ (0.85) and the shaded blue bar its 90% confidence interval.

Figure 4: Error about Aggregate Estimates : Leverage vs β targeting

Panel A: Error in Estimating $\log Y$ *loss*



Panel B: Error in Estimating $\log TFP$ *loss*



Note: In this figure, we report the distribution of misspecification errors on TFP and output losses across 4,000 alternative models. We simulate datasets from 4,000 alternative models. Each alternative model corresponds to the baseline model augmented in six different dimensions described in Section 5.3.3. We estimate TFP/output losses using the baseline (misspecified) model on these 4,000 datasets with two separate approaches: one estimation targets leverage; another estimation targets the reduced-form moment β. For each alternative model, the difference between these estimated losses and the actual losses in the true model corresponds to the misspecification error in the estimation. We report the distribution of these errors for output loss in Panel A and for TFP loss in Panel B.

APPENDIX (FOR ONLINE PUBLICATION)

This Appendix contains:

- the method used to solve and estimate the model (Section A)
- the methodology used to solve and estimate 4,000 alternative models in our exploration of misspecification (Section B)
- additional comparative static results in partial equilibrium designed to show that the model is well-behaved around the estimate (Section C).
- a description of the various adjustments to debt, total assets, physical capital, and investment that we use when we explore potential misspecification due to measurement issues in Section 4 (Section D).

A Solving the model and Estimation

This Appendix details the algorithms used to solve the model and estimate it. To estimate the model, one needs to find the set of parameters such that model-generated moments fit a pre-determined set of data moments. Because our model does not have an analytic solution, we need to use indirect inference to perform the estimation. Such inference is done in two steps:

- For a given set of parameters, we need to solve the model numerically, which means solving the Bellman problem (7) and obtain the policy function $S_{t+1} = (d_{t+1}, k_{t+1})$ as a function of $S_t = (d_t, k_t)$ and exogenous variables $X_t = (z_t, p_t)$.
- We then use this resolution technique to estimate the parameters that match best a set of moments chosen from the data. We explain the methodology (Simulated Method of Moments) and the numerical algorithm that we use to implement it.

A.1 Solving the model numerically

In this section, we describe how we numerically solve the firm's problem with given parameters.

A.1.1 Grid definition

In order to solve the model numerically, we need to discretize the state space $(S; X)$. Let us start with the two exogenous variables. The log productivity process z is discretized using the standard Tauchen method on 51 grid points. Log real estate prices are also an AR(1), discretized using the Tauchen method on 11 grid points. For both variables, we set the bounds of the grid at -2.5 and 2.5 standard deviations.

Capital choice is discretized over a range going from k_{min} to k_{max}. k_{min} is the smallest level of capital chosen by a firm without adjustment costs and financing constraints. For this particular case, we can solve the capital decision analytically. In most cases, this number should serve as a lower bound because adjustment costs would prevent firms from adjusting all the way down to this level; and financial constraints would push them to keep more capital as precautionary savings. Since we did not, however, establish this result analytically, we check that k_{min} is always "far enough" from the lowest simulated value of capital. Similarly, k_{max} is the capital stock chosen by unconstrained firms, without adjustment cost, facing the highest productivity level on the grid. Again, we expect this level to be above the upper bound of capital for a constrained firm with adjustment costs. We check that this is the case in our simulations. We then form an equally spaced grid for $\log$ capital between $\log k_{min}$ and $\log k_{max}$, with increment of $\log(1 + \delta/2)$. Thus, the capital grid is geometrically spaced using $(1 + \delta/2)$ as the multiplying coefficient, i.e. the nth point is equal to $k_{min} \times (1 + \delta/2)^n$ until k_{max}. Given that k_{min} and k_{max} are functions of productivity, the grid thus depends on the persistence ρ and volatility σ of log productivity. Larger persistence or volatility leads to a wider grid. In our preferred specification, capital evolves on a grid containing 270 points. We will take this number as a reference when we later discuss grid size, bearing in mind that, in fact, the capital grid is a function of parameter values.

Finally, the debt grid d_t is defined as a function of the amount of capital k_t. This adaptive feature of the debt grid comes from the fact that the amount of debt is bounded above by a function of capital: larger firms can borrow more. We restrict future period debt d' to the $\left[-4\bar{d}; \bar{d}\right]$ interval, where $\bar{d} = s\left((1-\delta)k + p_{max}h\right)$ and p_{max} is the maximum house price level. The grid interval is thus a function of the model parameters s but also ρ and σ via the grid of k. The upper bound is a natural consequence of the collateral constraint: the model imposes that it cannot be exceeded. The lower bound is somewhat arbitrary as there is in theory no upper bound as to how much cash the firm may decide to hold. We check that there is no accumulation of cash at this bound during the estimation process. Within this interval, the grid is geometrically spaced so that it is denser when debt becomes closer to the constraint, i.e. right below $\bar{d}$. We implement this by setting the nth grid point at $\bar{d}\left(1 - 0.001 \times e^{.3n}\right)$ until it reaches $-4\bar{d}$. Thus, the grid size for debt does not depend on parameters (in contrast to the capital grid size) and always has 29 points.

A.1.2 Bellman resolution algorithm

We solve the firm's problem using policy iteration. This algorithm is based on the fact that the value function is the solution of a fixed point problem generated by a contraction mapping.

Before starting to iterate, we compute profit flows $e(S, S'; X)$ using the production and cost functions, for all possible values of S and X on the grid. We set e to "missing" when $(S, S'; X)$ are such that $e < 0$ – the no equity issuance constraint is violated, or when the borrowing constraint is violated. Profits are only defined when both financing constraints are satisfied.

To initiate the process, we start with the value function $V_0\left(S; X\right) = 1$. We then look for the policy function $(k'_0, d'_0) = P_0(S; X)$ which solves:

$$P_0(S; X) = \textbf{argmax}_{S'}\{e(S, S'; X) + \frac{1}{1+r}\}$$

for each state of $(S; X)$. Then, we iterate the following loop (where $n \geq 1$ denotes the step in the loop):

1. Start from $(k'_{n-1}, d'_{n-1}) = P_{n-1}(S; X)$, the policy function obtained from the previous round; and $V_{n-1}\left(S; X\right)$, the value function obtained from the previous round. For every point $(S; X)$ on the grid, we compute the value function V_n that satisfies:

$$V_n(S; X) = e(S, P_{n-1}(S; X); X) + \frac{1-d}{1+r}\mathbb{E}_{X'}\left[V_{n-1}(P_{n-1}(S; X'); X')|X\right] + \frac{d}{1+r}\left(k'_{n-1} - (1+\tilde{r}_t)d'_{n-1}\right)$$

2. We then use the new value function V_n and compute the optimal policy given

this value function ($P_n(S;X)$):

$$P_n\left(S;X\right) = \textbf{argmax}_{S'}\{e(S,S';X) + \frac{1-d}{1+r}\mathbb{E}_{X'}\left[V_n(S';X')|X'\right] + \frac{d}{1+r}\left(k' - (1+\tilde{r}_t)d'\right)\}$$

3. We stop when $P_n = P_{n-1}$.

Thanks to the contraction mapping theorem, we are guaranteed to find a good approximation of the value function $V(S;X)$ and the policy function $S' = P(S;X)$ defined over the grid. The computationally costly step is the determination of the policy function in step 2 with respect to S'. This consists of $29 \times 270 \times 51 \times 11 = 4,392,630$ optimizations of vectors with $29 \times 270 = 7,830$ points. This is where parallelization achieved through a GPU accelerates the process. For the range of parameters we explore, we typically solve the model in about 2 minutes with a GPU (Nvidia K80), compared to several hours with a CPU. What prevents us from having a finer grid is the RAM of the GPU, since the computer needs to create the maximand in step 2, a $29 \times 270 \times 29 \times 270 \times 51 \times 11 \approx 34$ billion numbers array.

The above algorithm is the standard policy function iteration algorithm. We make two adjustments to adapt it to our setting. First, in order to reduce computing time, we first solve the model with a coarser grid and then solve it again on the grid described above. To define this coarser grid, we divide the resolution of the control (k and d) grids by two. This divides computing time by four in the first step but only gives us the value and policy functions on the coarser grid. We then re-run the algorithm on the finer grid with the "coarser" policy and value functions as a starting point. Convergence occurs much more quickly.

The other adjustment is related to the treatment of missing values, which in our set-up occur when one of the two financing constraints is violated (i.e. the no-equity constraint or the collateral constraint). Without modification, the policy iteration algorithm does not behave well in the presence of missing values. This is because, for some given value functions V_{n-1}, there may exist some (S,X) for which there is no acceptable policy S'. In this case, the optimal policy function $P_n(S,X)$ is not defined everywhere on the grid (note (S_0,X_0) such states for which the policy is not defined). When this happens, the next iteration value $V_n(S;X)$ is non-defined for all (S,X), which leads with non-zero probability to states (S_0,X_0). As we iterate, missing value progressively spread to the entire grid and the algorithm is blocked. To solve this problem, we modify step 1. of the algorithm by requiring that $V_n(S;X)$ replaces $V_{n-1}(S;X)$ if and only if $V_n(S;X)$ is nonmissing. This prevents missing values from spreading to the entire grid of states $(S;X)$.

A.2 Estimation

We now proceed to estimate the parameters (s,c,ρ,σ,e) for which the model best matches a predefined set of moments (we experiment with a different set of moments and models in the main text).

A.2.1 Estimation method: SMM

We estimate the key parameters of the model by simulated method of moments (SMM), which minimizes the distance between moments from real data and simulated data. Let us call $\mathbf{m}$ the vector of moments computed from the actual data, and let us call Ω the moments generated by the model with parameters Ω. The SMM procedure searches the set of parameters that minimizes the weighted deviations between the actual and simulated moments,

$$(\mathbf{m} - \widehat{\mathbf{m}}(\boldsymbol{\Omega}))' \mathbf{W} (\mathbf{m} - \widehat{\mathbf{m}}(\boldsymbol{\Omega})) \tag{15}$$

We detail the various components of our implementation in the following sections.

A.2.2 Empirical moments m and Weight matrix W

The empirical moments are computed in a simple way, and the definitions are given in the main text, in Section 3.3.

The weight matrix $\mathbf{W}$ adjusts for the fact that some moments are more precisely estimated than others. It is computed as the inverse of the variance-covariance matrix of actual moments estimated by bootstrap with replacement on the actual data. To compute the elements of this matrix, we repeat 100 times the following procedure. Using our dataset, we draw, with replacement, N firms with their entire history where N is the number of firms in the sample (we use the bsample command in Stata, clustered at the firm level). We then compute the moments and store them. Once we have performed this procedure 100 times, we compute the empirical variance-covariance matrix of the moments and invert it.

A.2.3 Model-generated moments m

Once we have solved the model for a given set of parameter $\boldsymbol{\Omega}$ (Appendix A.1), we need to simulate data in order to compute the simulated moments. We simulate a balanced panel of 1,000,000 firms over 100 years, and only keep the last 10 years to ensure each firm has reached steady-state. For each firm, we simulate a path of log productivities z_{it} and a path of log real estate prices p_{it}. This makes the variability of real estate prices larger than in the data, where prices only vary at the MSA level. Recall however that our objective in this simulation is not to replicate the variability of the data, but ideally to estimate model-generated moments. If we had closed forms for the model, we could measure these moments without infinite precision. The problem here comes from the fact that we cannot directly compute these moments but have to "estimate" them. Ideally, we would want to generate an infinitely large simulated dataset in order to compute the model-generated moments exactly, but computational constraints make it infeasible. 1,000,000 firms over 100 years already generate arrays with 100m entries. Allowing real estate prices to vary at the firm-level is a way to make sure the sensitivity to prices model-generated moments are estimated as well as possible.

A.2.4 Optimization algorithm

We now have all the ingredients necessary to compute the objective function (15). In this section, we explain how to minimize it. Since in our most preferred specification we have 5 parameters, we need to make sure that we are indeed reaching a global minimum. We do this by implementing the following two-step procedure, which follows Guvenen et al. (2014):

- We generate 1,000 quasi-random vectors of parameters Ω taken from a Halton sequence. The Halton sequence is a deterministic sequence of numbers that has the property of covering the parameter space evenly. For each of these parameters, we solve the model to obtain the policy function, simulate a dataset, compute the moments, and therefore the distance to data moments (15).

- We then use the lowest points (in terms of the objective function) as starting points for minimization. We iterate on the following loop. We begin with parameter estimate $\hat{\Omega}_1$ for which the objective function is the lowest. We then use the Nelder-Mead method (command *fminsearch* in Matlab) to perform local optimization starting from this point. We then compute the objective function O_1. We then move to the second-lowest parameter estimate ($\hat{\Omega}_2$) and compute the objective function O_2. We iterate on this and stop as soon as $O_n = 0$. Among the lowest parameters, a large fraction typically leads to the same parameters for which the objective function is equal to 0. This gives an indication that our objective function is well-behaved.

There are no general theoretical results arguing that this technique dominates other popular algorithms adapted for large dimension optimization. In our setting, however, we found that the genetic algorithm and simulated annealing were much slower at converging. Also, this approach allows to "control" the smoothness of the objective function. For instance, within the lowest 20 parameters isolated after step 1., it would be worrisome if minimizations starting from each of these parameters gave inconsistent parameters. On the contrary, they tend to be very consistent. The only cases where convergence goes to the alternative choice of parameters than the one we present are cases where the objective function is much bigger than zero (i.e. other local optima). Finally, the best argument in favor of our selected estimates is the well-behaved comparative statics we present in Appendix C.

A.2.5 Standard errors

To compute standard errors, we use the standard formula for the variance-covariance matrix of parameter estimates $\hat{\Omega}$ in SMM estimation:

$$\left(J'W^{-1}J\right)^{-1} + \left(J'\tilde{W}^{-1}J\right)^{-1}$$

which is composed of two terms. The first term corresponds to the error coming from the estimation of data moments. W is the variance-covariance matrix of data moments (which we use as weighting matrix in our SMM and estimate by bootstrap –

see the previous section). Intuitively, the data induce little estimation error if W is small (the moments are precisely estimated). The second term comes from the noise we face when estimating the simulated moments: $\tilde{W}$ is the variance-covariance matrix of moments in simulated moments. It is an object that can also be estimated by bootstrap like W. Intuitively, the more precise the estimation of simulated moments (for instance, because the simulated dataset is very large), the smaller this term. Last, J is the Jacobian matrix around the SMM estimate. The bigger it is, the more sensitive moments are to parameters. And when this is the case, the estimation is more precise. With some abuse of language, economists tend to say that the model is well identified (locally, around $\hat{\Omega}$) when the coefficients in J are large. It is more accurate to say that the model is precisely estimated. We have a discussion like this in the main text (Section 3.4) and report the log Jacobian matrix (the matrix of elasticities) in Table 1.

Ideally, we would want to try to minimize the error coming from the imprecise estimation of the model-generated moments. This estimation is imprecise because we do not simulate an infinitely large dataset. If we did, we would have $\tilde{W} = 0$ and the second term in equation (A.2.5) would be zero. In Appendix A.2.3, we make sure that $\tilde{W}$ is as small as possible (by maximizing the number of observations and having one price path per firm), but, in practice, the sensitivity of investment to real estate prices is still estimated with significant error, which forces us not to neglect the second term in equation (A.2.5).

In practice, we compute W as described in Section A.2.2. $\tilde{W}$ is similarly computed by bootstrapping the simulated sample using SMM-estimated parameters. We start with simulated data (100,000 firms followed over 100 years each). We then draw 100,000 firms from this sample *with replacement*. We compute the moments and repeat this procedure 100 times. Using these 100 sets of model-generated moments, we then compute the variance-covariance matrix $\tilde{W}$. We estimate J using the following technique. For each parameter j, we note $\hat{\Omega}_j$ the SMM estimate. We then simulate the model, holding the other parameters constant, for ω_j^+ immediately above $\hat{\omega}_j$ and ω_j^- immediately below. We use a 10-grid point scale as in the figures of Appendix C. We compute model-generated moments m^+ and m^- associated with these two variants. The moment gradient $\frac{m^+ - m^-}{\omega_j^+ - \omega_j^-}$ is the j^{th} column of our estimate the J matrix.

B Building the Dataset of Misspecification Errors

B.1 Auxiliary model

We perform our Monte Carlo analysis using a dedicated and accelerated SMM procedure. This procedure relies on an auxiliary model that captures the relationships between each simulated moment and the vector of parameters $\mathbf{\Omega} = (\rho, \sigma, s, c, e, h)$ estimated by the econometrician. Specifically, we assume that each moment k is a cubic polynomial function f_k of the parameters, including cross terms. So the auxiliary model is a set of polynomial equations. Hence, if we know the coefficients of these polynomial functions, we can predict each simulated moment and the SMM error function using the auxiliary model. The advantage of this approach is that the computing time of the auxiliary model is shorter by several orders of magnitude.

The first step of this procedure is to train the auxiliary model. To do so, we generate a training sample using using the real structural model. We draw 20,000 vectors of parameters $\mathbf{\Omega}_i$ and generate the simulated moments $\widehat{\mathbf{m}}(\mathbf{\Omega}_i)$ by simulating the true structural model under our baseline specification and calibration. By doing so, we obtain a dataset with 20,000 observations, which contain the vector of generating parameters $\mathbf{\Omega}_i$ and the vector of simulated moments $\widehat{\mathbf{m}}(\mathbf{\Omega}_i)$. The vectors $\mathbf{\Omega}_i$ are drawn from a quasi-random (Halton) sequence with $\rho \in [.50.95]$, $\sigma \in [.08.17]$, $s \in [.011]$, $h \in [.05.4]$, $c \in [0.04]$ and $e \in [0.40]$.

Our goal is to use this training dataset to predict the vector $\widehat{\mathbf{m}}(\mathbf{\Omega}_i)$ using polynomial approximations instead of the full structural model. We test this approach as follows.

1. Take a vector $\mathbf{\Omega}_j$ and generate the vector of simulated moments $\widehat{\mathbf{m}}(\mathbf{\Omega}_j)$ using the full structural model.

2. Using the training dataset, compute the SMM error function for each observation i, that is:
$$\left(\widehat{\mathbf{m}}\left(\mathbf{\Omega}_j\right) - \widehat{\mathbf{m}}\left(\mathbf{\Omega}_i\right)\right)' \mathbf{W} \left(\widehat{\mathbf{m}}\left(\mathbf{\Omega}_j\right) - \widehat{\mathbf{m}}\left(\mathbf{\Omega}_i\right)\right)$$

3. For each moment k, estimate the generating funciton f_k with a weighted cubic polynomial regression, using the invert of the SMM error function as regression weights.

4. For each moment k, use the estimated polynomial $\hat{f}_k$ to predict the simulated moments $\hat{f}_k(\Omega_j)$.

By repeating this procedure 100 times, we get a sense of the quality of our auxiliary model. Specifically, we find that the auxiliary model predicts each simulated moment generated by the true structural model with an R^2 above 0.97. This out of sample performance gives us confidence that we can use this approach to perform our Monte Carlo analysis.

B.2 Monte Carlo procedure

The Monte Carlo procedure works as follows. We start by drawing 4,000 vectors of parameters $\mathbf{\Omega}_i = (\rho_i, \sigma_i, s_i, c_i, e_i, h_i)$ and 6 supplementary parameters $\Theta = (I_i, U_i, \sigma_{u,i}, \kappa_i, d_{0,i}, \tau_i)$. To do so, we use a quasi-random (Halton) sequence with $\rho \in [.80\ ,\ .95]$, $\sigma \in [.10\ ,\ .15]$, $s \in [.10\ ,\ .7]$, $h \in [.10\ ,\ .3]$, $c \in [0\ ,\ .03]$, $e \in [0\ ,\ .30]$, $I \in [0\ ,\ .4]$, $U \in [0\ ,\ .4]$, $\kappa \in [-.3\ ,\ .3]$, $d_0 \in [.0\ ,\ .4]$, $\sigma_u \in [0.\ ,\ 0.03]$ and $\tau \in [1/3 - .075\ ,\ 1/3 + .075]$.

Then for each random draw we:

1. Solve the true, correctly specified model and simulate the moments. Compute the true TFP and output losses caused by financial constraints.

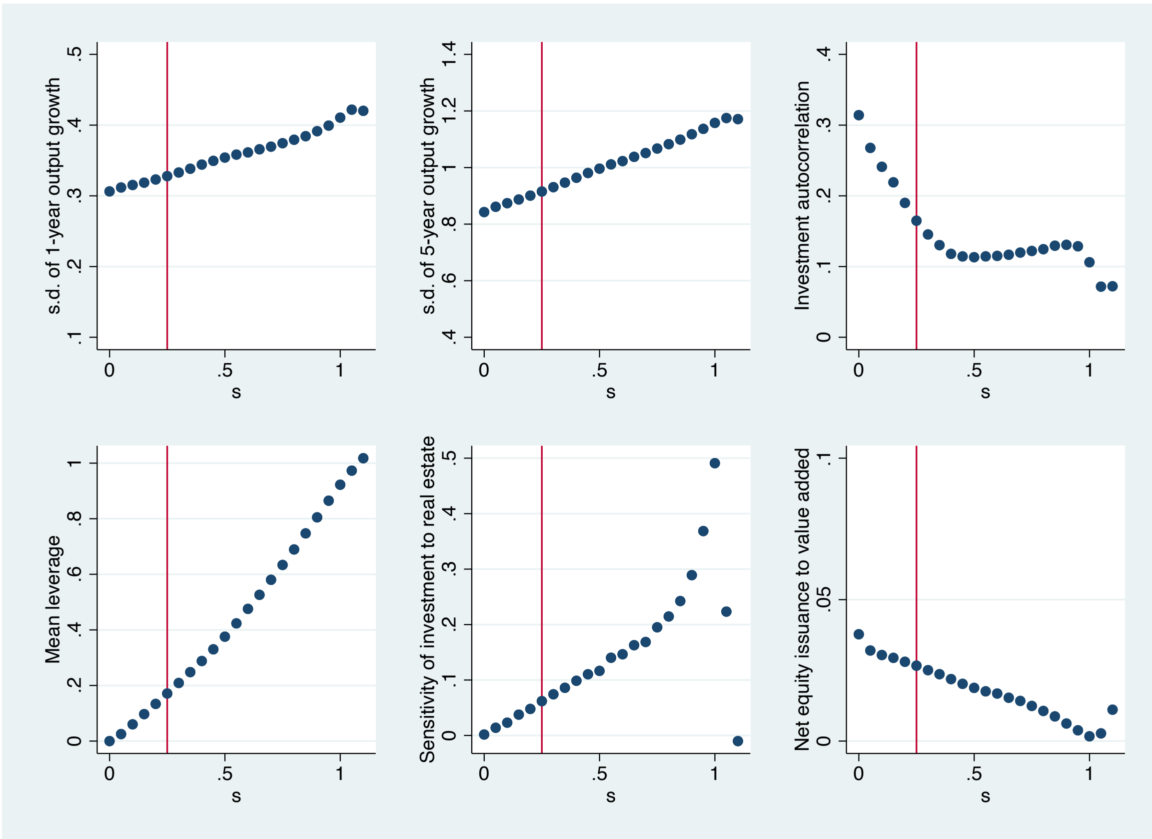
2. Set $\Theta = (0, 0, 0, 0, 0, .33)$ as in our baseline specification and estimate the vector $\mathbf{\Omega}$ by SMM. First, we target our investment sensitivity moment. Because we need to estimate the model thousands of times, we use the auxiliary model described in section B.1 and the global optimization routine described in section A.2.4.

3. Compute the TFP losses and output losses implied by the structural estimation of the misspecified model.

4. Repeat the last two steps but target leverage instead of the investment sensitivity moment.

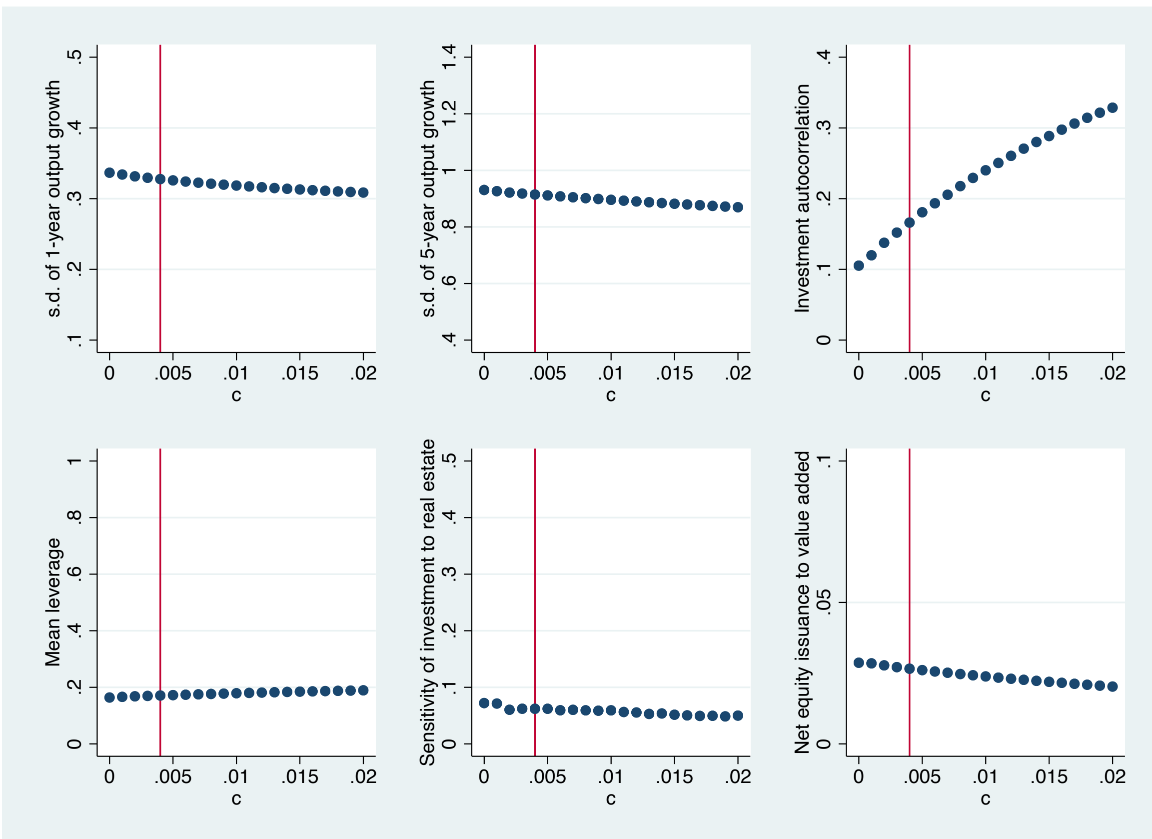
C Additional figures

Figure C.1: Sensitivity of moments to pledgeability s



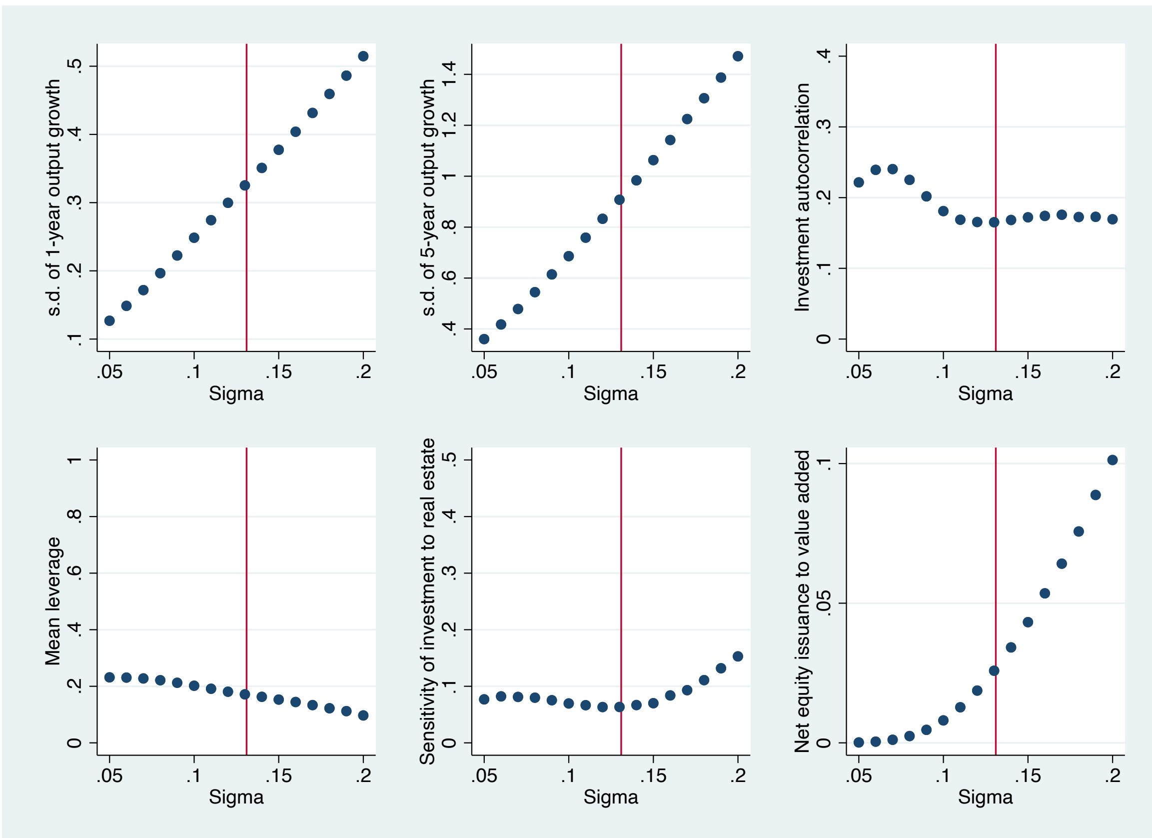
Note: In this figure, we set all estimated parameters (s, c, ρ, σ, H and e) at their SMM estimate in our preferred specification – as per column 3, Panel A in Table 2. We fix w and Q at their reference levels: $w = 0.03$ and $Q = 1$. We then vary s from 0 to 1. For each value of s that we choose, we solve the model, simulate the data, and compute four target moments, plus the average leverage ratio and the sensitivity of debt issuance to real estate value. Each panel corresponds to one moment. The red vertical line corresponds to the SMM estimate of s.

Figure C.2: Sensitivity of moments to adjustment costs c



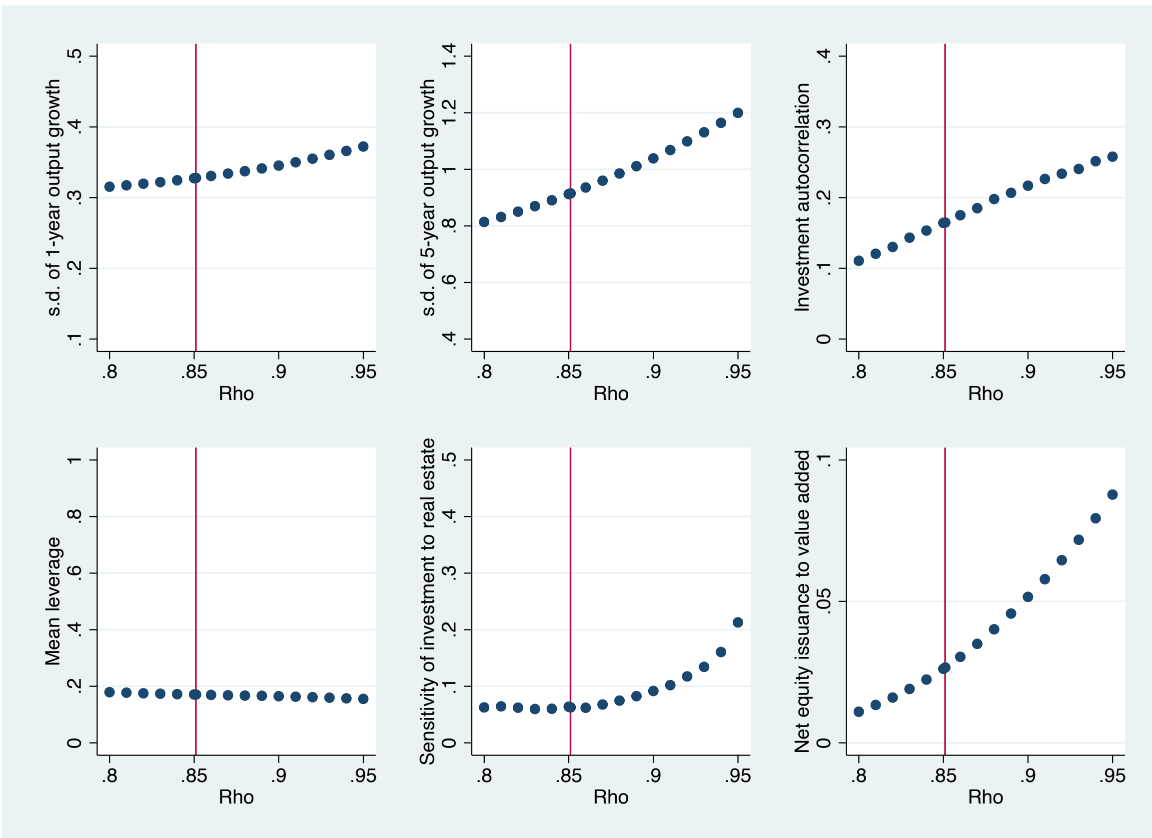
Note: In this figure, we set all estimated parameters (s, c, ρ, σ, H and e) at their SMM estimate in our preferred specification – as per column 3, Panel A in Table 2. We fix w and Q at their reference levels: $w = 0.03$ and $Q = 1$. We then vary c from 0 to 0.02. For each value of c that we choose, we solve the model, simulate the data, and compute four target moments, plus the average leverage ratio and the sensitivity of debt issuance to real estate value. Each panel corresponds to one moment. The red vertical line corresponds to the SMM estimate of c.

Figure C.3: Sensitivity of moments to productivity volatility σ



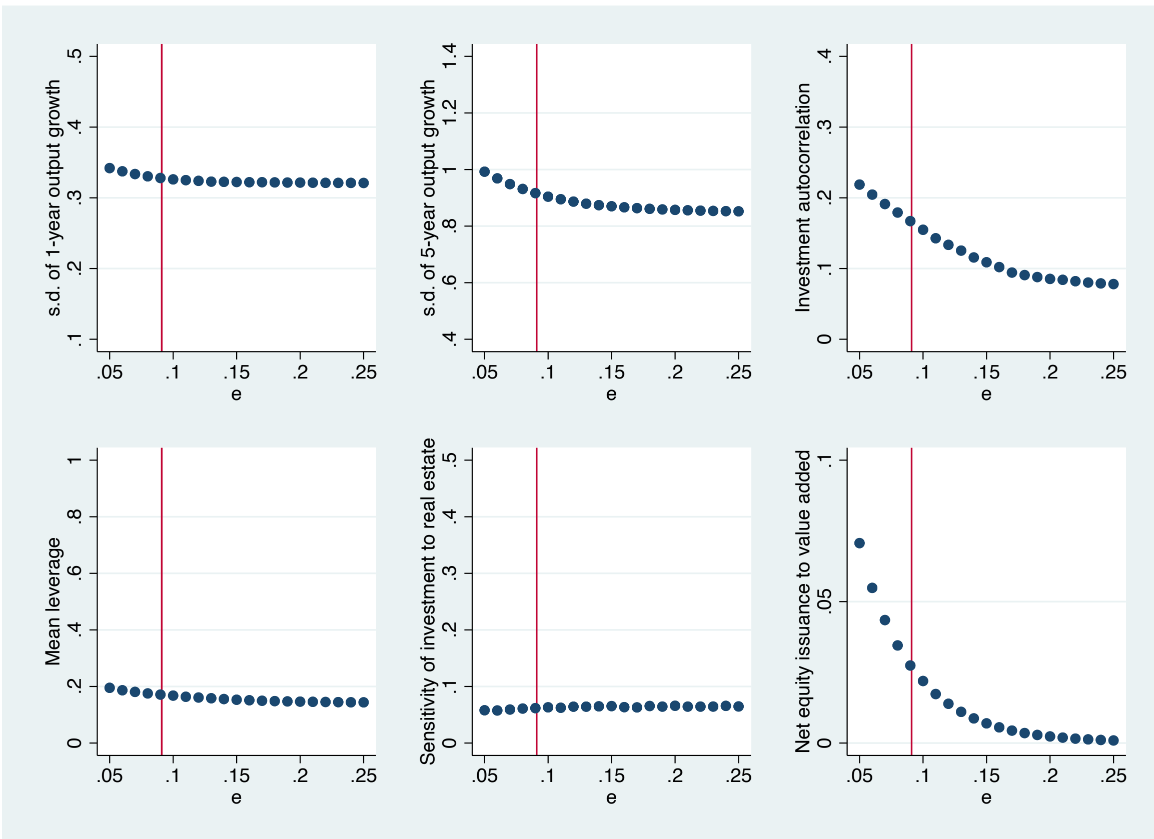
Note: In this figure, we set all estimated parameters (s, c, ρ, σ, H and e) at their SMM estimate in our preferred specification – as per column 3, Panel A in Table 2. We fix w and Q at their reference levels: $w = 0.03$ and $Q = 1$. We then vary σ from 0 to .2. For each value of σ that we choose, we solve the model, simulate the data, and compute four target moments, plus the average leverage ratio and the sensitivity of debt issuance to real estate value. Each panel corresponds to one moment. The red vertical line corresponds to the SMM estimate of σ.

Figure C.4: Sensitivity of moments to productivity persistence ρ



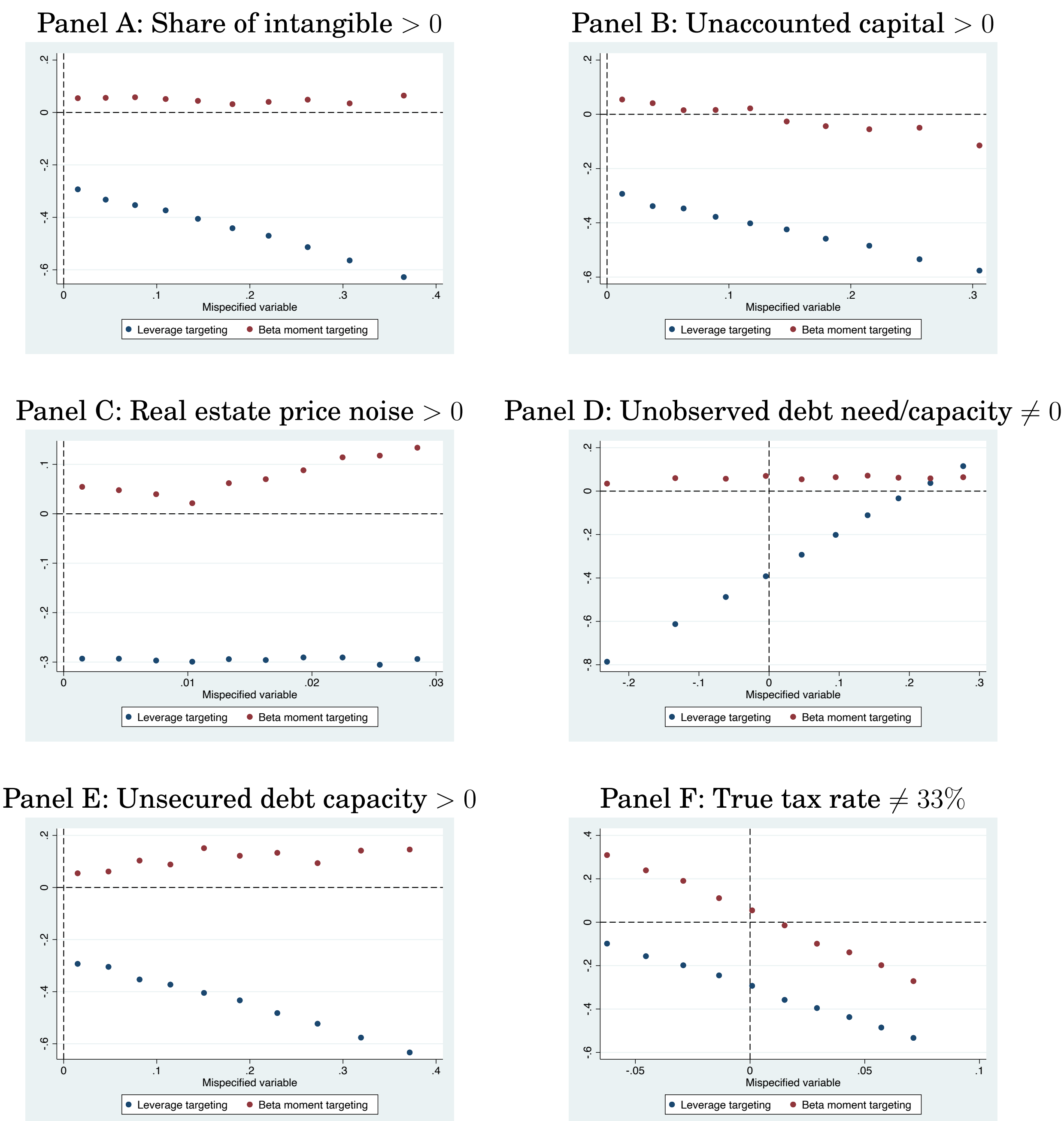
Note: In this figure, we set all estimated parameters (s, c, ρ, σ, H and e) at their SMM estimate in our preferred specification – as per column 3, Panel A in Table 2. We fix w and Q at their reference levels: $w = 0.03$ and $Q = 1$. We then vary ρ from 0.8 to 0.95. For each value of ρ that we choose, we solve the model, simulate the data, and compute four target moments, plus the average leverage ratio and the sensitivity of debt issuance to real estate value. Each panel corresponds to one moment. The red vertical line corresponds to the SMM estimate of ρ.

Figure C.5: Sensitivity of moments to equity issuance cost e



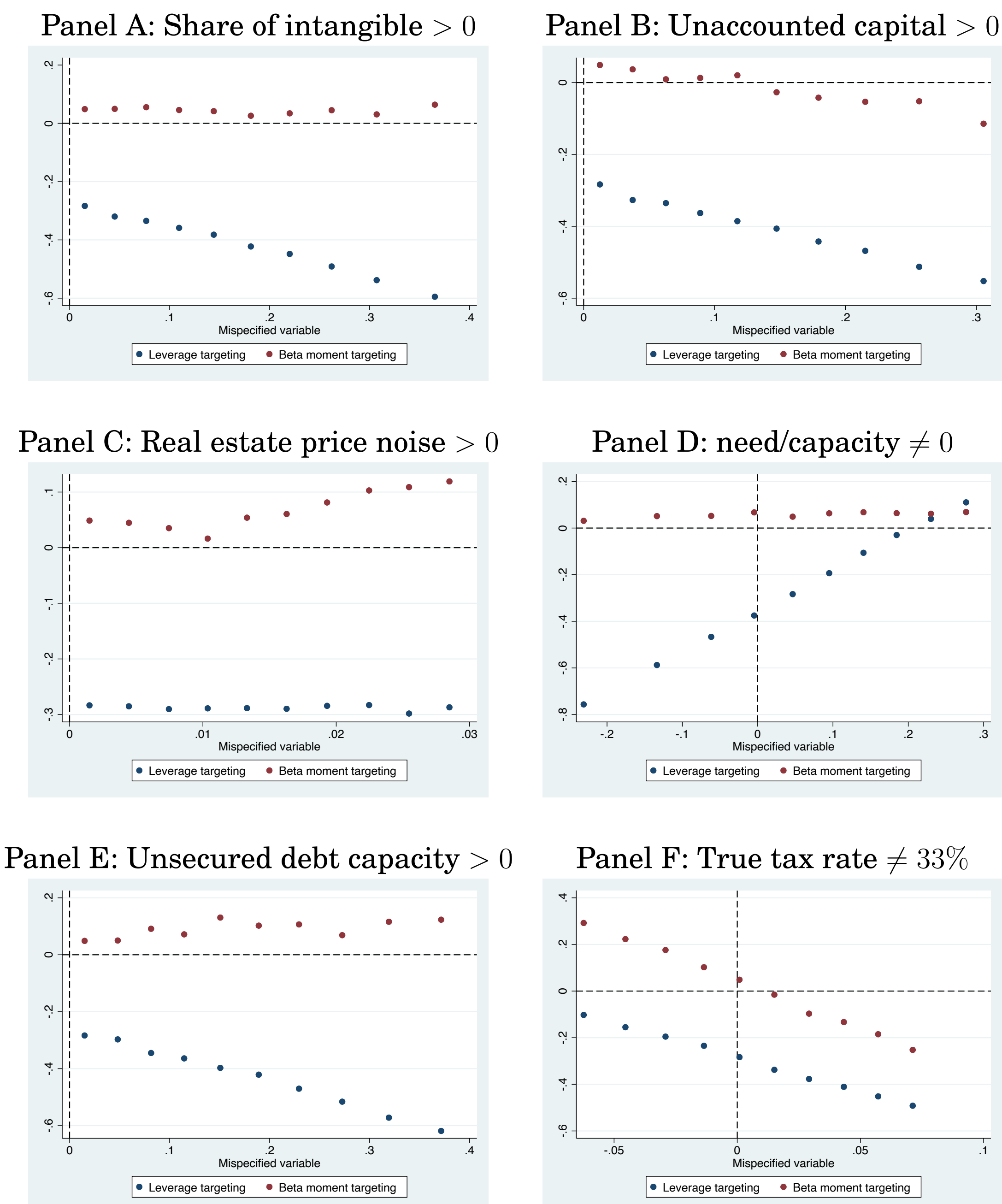
Note: In this figure, we set all estimated parameters (s, c, ρ, σ, H and e) at their SMM estimate in our preferred specification – as per column 3, Panel A in Table 2. We fix w and Q at their reference levels: $w = 0.03$ and $Q = 1$. We then vary e from 0.05 to 0.25. For each value of e that we choose, we solve the model, simulate the data, and compute four target moments, plus the average leverage ratio and the sensitivity of debt issuance to real estate value. Each panel corresponds to one moment. The red vertical line corresponds to the SMM estimate of e.

Figure C.6: Misspecification and TFP loss estimation error, leverage vs β targeting



Note: In this figure, we report the effect of misspecification on the estimation of TFP loss. On our sample of 4,000 simulations, we first regress misspecification errors on 10 dummies for each decile of the six parameters governing misspecification (Θ). We do this for the two sets of misspecification errors obtained using either inference based on leverage or inference based on the reduced-form coefficient β. We then plot the estimated coefficient for each of these dummy variables in 6 separate panels. All y-axes are on the same scale except for Panel D, which is on a slightly larger one.

Figure C.7: Misspecification and output loss estimation error, leverage vs β targeting



Note: In this figure, we report the effect of misspecification on the estimation of output loss. On our sample of 4,000 simulations, we first regress misspecification errors on 10 dummies for each decile of the six parameters governing misspecification (Θ). We do this for the two sets of misspecification errors obtained using either inference based on leverage or inference based on the reduced-form coefficient β. We then plot the estimated coefficient for each of these dummy variables in 6 separate panels. All y-axes are on the same scale except for Panel D, which is on a slightly larger one.

D Leverage and β with misspecification

In this section, we explain in detail the adjustments we do to calculate leverage and estimate β under the various sources of misspecification considered in Table 4. Line 1 corresponds to our baseline estimate for β and our baseline leverage.

In line 2, debt is unchanged, but total assets are now the sum of book assets and the off-balance sheet intangible capital in Peters and Taylor (2017). For the estimation of β, the dependent variable is the ratio of capital expenditures plus R&D expenditures plus 30% of SG&A (Peters and Taylor (2017)), normalized by the sum of lagged tangible assets and lagged intangible capital ($k^{\text{intangible}}$ from Peters and Taylor (2017)). In terms of explanatory variables, $REValue_{it}$ is normalized by lagged tangible assets and lagged intangible capital ($k^{\text{intangible}}$ from Peters and Taylor (2017)); Cash flows add back $(1 - \tau)$ of the expenditures in intangible capital (capital expenditures plus R&D expenditures plus 30%) and are normalized by lagged tangible assets and lagged intangible capital (Peters and Taylor (2017)). We use a tax rate of $\tau = 30\%$. We use the total Q in Peters and Taylor (2017) as our measure of Q.

In line 3, we calculate leverage by adding leases to both the numerator and the denominator of our baseline measure. We calculate leases following an approach similar to Rampini and Eisfeldt (2009). For each firm-year, we compute l, the ratio of operating lease rentals (lagged COMPUSTAT item mrc1) to on-balance sheet capital rental (depreciation plus 10% of total assets (COMPUSTAT item at)). We then compute leases as $l \times at$. To estimate β, the dependent variable is the ratio of capital expenditures plus the yearly difference in leases, normalized by the sum of lagged tangible assets and lagged leases. In terms of explanatory variables, $REValue_{it}$ and cash-flows are normalized by the sum of lagged tangible assets and lagged leases. Our market-to-book ratio is similar to the baseline, except that the denominator is now the sum of total assets and leases.

In line 4, we calculate leverage by adding the present value of lease commitments (PV(lease)) to the net debt used in our baseline measure. To calculate PV(lease), we start from the next five years commitments (mtr1-5), spread expected commitment (mrtca) over these 5 years equally, and calculate the present value of these commitments at a 10% discount rate. The estimate for β corresponds to the baseline estimate since the measurement of capital is similar to the baseline.

In line 5, we calculate leverage by adding account payables (ap) to the net debt used in our baseline measure. To estimate β, the dependent variable is now the ratio of capital expenditures plus the yearly difference in current assets, normalized by the sum of lagged tangible assets and lagged current assets. $REValue_{it}$ and cash-flows are normalized by the sum of lagged tangible assets and lagged current assets. Our market-to-book ratio is similar to the baseline.

In line 6, we calculate leverage by using the economic value of the physical capital stock instead of net PP&E in the denominator. To calculate the capital stock, we start with net PP&E (COMPUSTAT item ppent) in the first year the firm appears in COMPUSTAT (or in 1981 if the firm appears prior to 1981). Every year, we then add capital expenditures (COMPUSTAT item capx), subtract sale of property (COMPUSTAT item SPPE) and depreciate the previous year capital stock at a 6% rate, which is typical in the macro-finance literature (e.g. Midrigan and Xu (2014)). Leverage is then calculated as the ratio of net debt to total assets minus PP&E plus the physical capital stock. To estimate β, the dependent variable is the ratio of capital expenditures normalized by the lagged physical capital stock. $REValue_{it}$ and cash-flows are normalized by the lagged capital stock. The market-to-book ratio is similar to the baseline, except that the denominator is total assets minus net PP&E plus the capital stock calculated with the inventory method.

In line 7, we combine all these adjustments. We calculate net debt using the baseline measure, to which we add leases and accounts payables. Total assets are given by assets minus PP&E plus the physical capital stock, to which we add leases, and the off-balance sheet part of intangible capital ($K_{int}^{\text{off b-s}}$). Leverage is the ratio of these two variables. To estimate β, we use the following dependent variable:

$$Y_t = \frac{\text{capital expenditures}_t\text{+ R\&D expenditures}_t\text{+30\%.SG\&A}_t\text{+(leases}_t\text{-leases}_{t-1}\text{)+(Current}_t\text{-Current}_{t-1}\text{)}}{K_{t-1} + \text{current}_{t-1} + \text{leases}_{t-1} + k_{t-1}^{\text{intangible}}},$$

where K_{t-1} corresponds to the lagged capital stock calculated using the perpetual inventory methodology described above. $REValue_{it}$ and cash-flows are normalized by the same denominator as the one in Y_t. As in line 2, cash-flows include (1-τ)(R&D expenditures$_t$+30%.SG&A$_t$). Finally, our measure of Q is as in the baseline, but uses as denominator: (total assets + capital stock - PP&E+leases+ k_{int}^{offbs}+current), where k_{int}^{offbs} is the off-balance sheet intangible capital from Peters and Taylor (2017).